Measurement-based QIP with *minimal resource* graph states

Imoto Lab, Osaka University,
Wed. November 28th 2007

Mark Tame

QTeQ - Quantum Technology at Queen's
Queen's University, Belfast
www.qteq.info

Queen's University Belfast

QTeQ QUANTUM TECHNOLOGY at QUEEN'S

JSPS Japan Society for the Promotion of Science

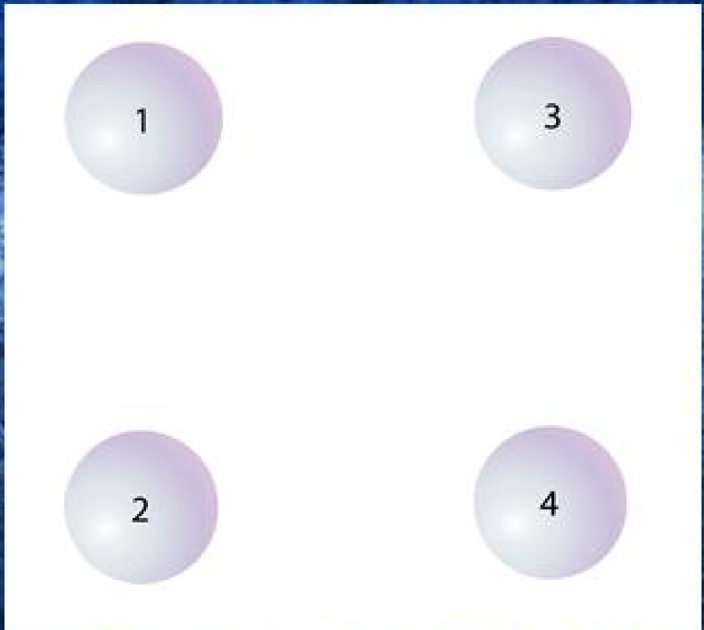
- Introduction to Graph States and their properties
- Weighted Graph States
- Measurement-based QIP with minimal resource graphs
- Fusing weighted graphs
- Summary and Outlook


Q: What is a graph state?

Introduction to Graph States and their properties

A simple graph state

1) Preparation of $|+\rangle$



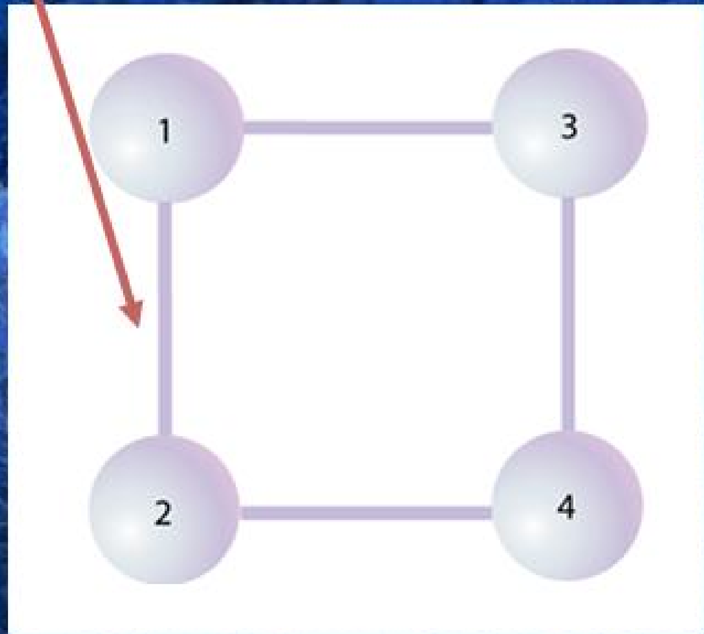
$| \psi \rangle = |+\rangle |+\rangle |+\rangle |+\rangle$

$$|\pm\rangle_j = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)_j$$

CZ: $|0\rangle |0\rangle \;\to\; |0\rangle |0\rangle$
$|0\rangle |1\rangle \;\to\; |0\rangle |1\rangle$
$|1\rangle |0\rangle \;\to\; |1\rangle |0\rangle$
$|1\rangle |1\rangle \;\to\; - |1\rangle |1\rangle$

2) Application of CZ's

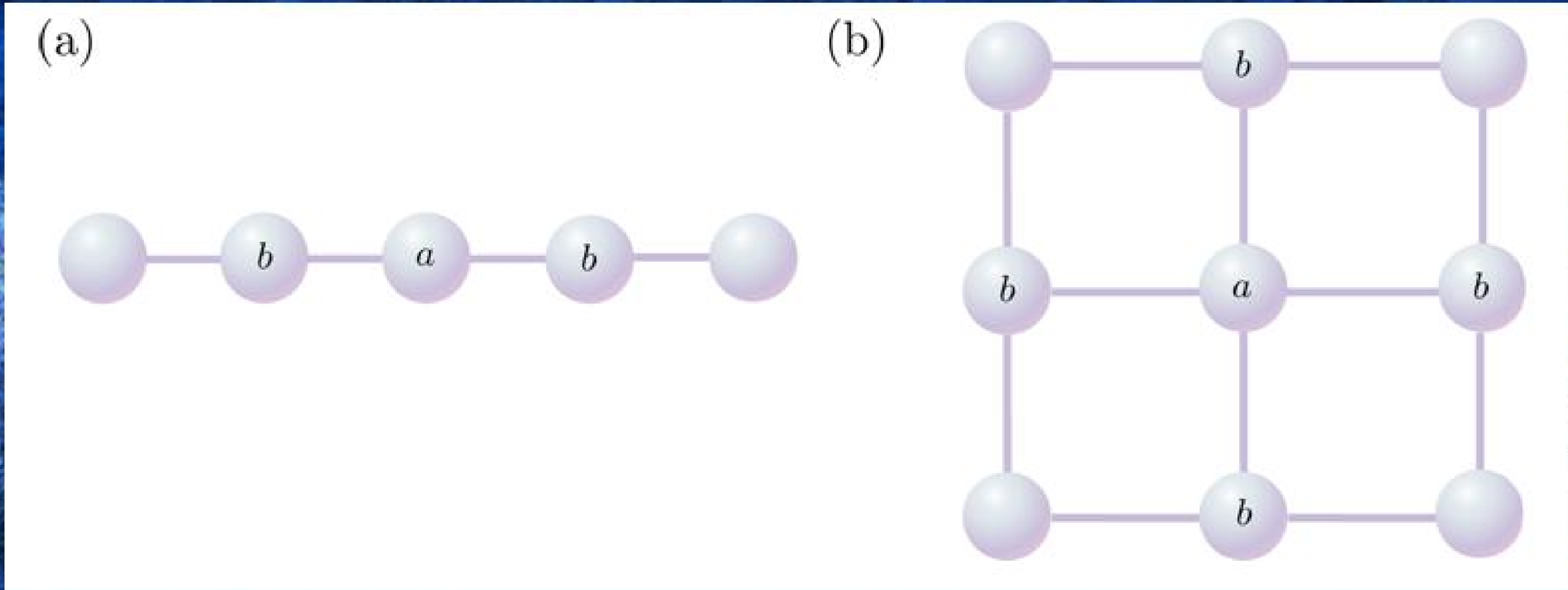
$$CZ = |0\rangle_a\langle 0| \otimes \mathbb{1}_c + |1\rangle_a\langle 1| \otimes \sigma_{z,c}$$



$| \psi \rangle = 1/4(|+\rangle |+\rangle |+\rangle |+\rangle + |+\rangle |-\rangle |+\rangle |-\rangle$

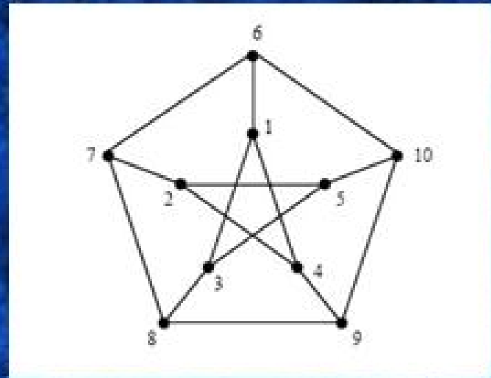
$+ |-\rangle |+\rangle |-\rangle |+\rangle - |-\rangle |-\rangle |-\rangle |-\rangle)$

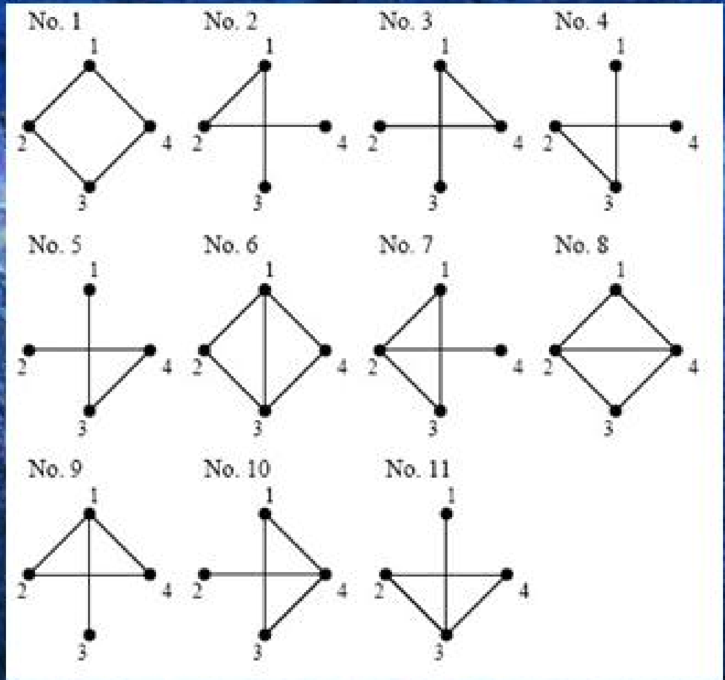
Introduction to Graph States and their properties

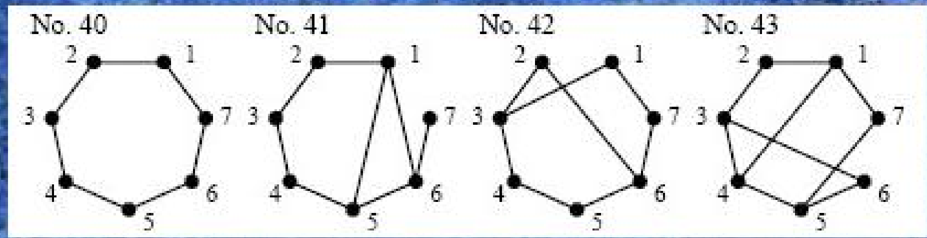


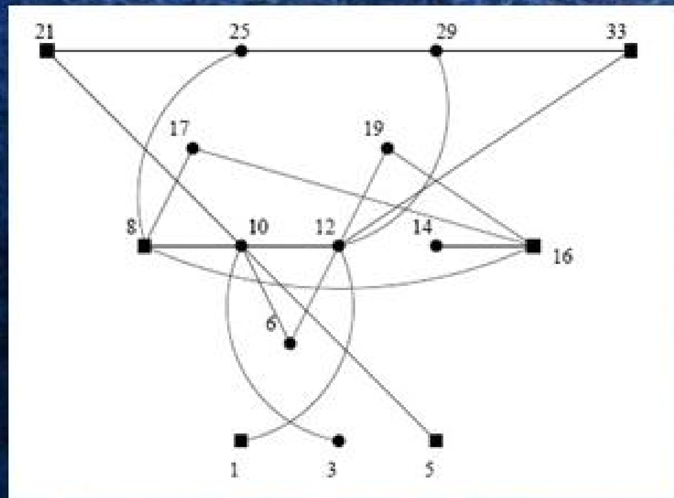
Introduction to Graph States and their properties

More complicated graph states







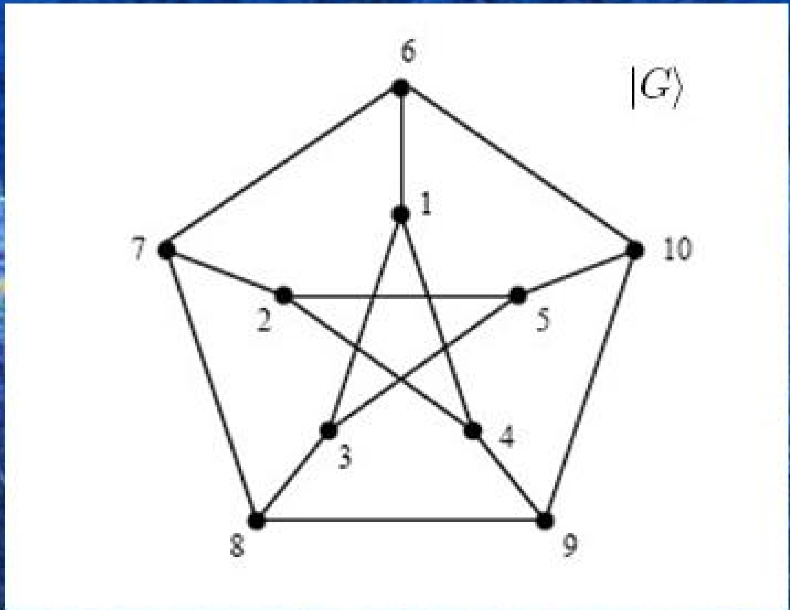


-Hein et al., *Proceedings of the International School of Physics "Enrico Fermi" on "Quantum Computers, Algorithms and Chaos"*, Varenna, Italy, July, 2005; also at quant-ph/0602096

-Hein, Eisert and Briegel, PRA (2004)

Introduction to Graph States and their properties

General Properties



Petersen Graph

-Hein et al., quant-ph/0602096

$$G = (V, E)$$

$$V = \{1, \ldots, N\} \qquad E \subset [V]^2$$

$$\textit{adjacency matrix } \mathbf{\Gamma}_G = \mathbf{\Gamma}$$

$$\mathbf{\Gamma}_{ab} = \begin{cases} 1, & \text{if } \{a,b\} \in E, \\ 0 & \text{otherwise.} \end{cases}$$

Correlations

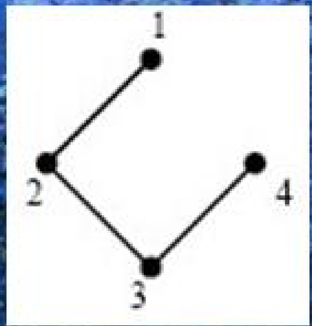
$$K_G^{(a)}|G\rangle = |G\rangle \qquad K_G^{(a)} = \sigma_x^{(a)} \prod_{b \in V} (\sigma_z^{(b)})^{\Gamma_{ab}}.$$

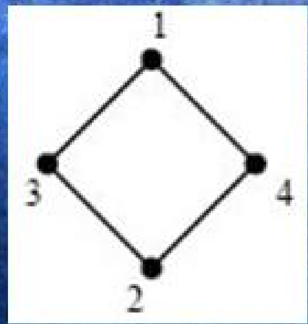
$$N_a := \{b \in V \mid \{a,b\} \in E\} \qquad K_G^{(a)} = \sigma_x^{(a)} \prod_{b \in N_a} \sigma_z^{(b)}.$$

Introduction to Graph States and their properties

***Entanglement** in graph states is not always what it seems...*

Local Unitary (LU) equivalent

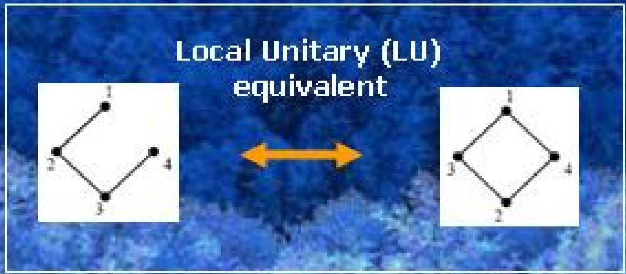




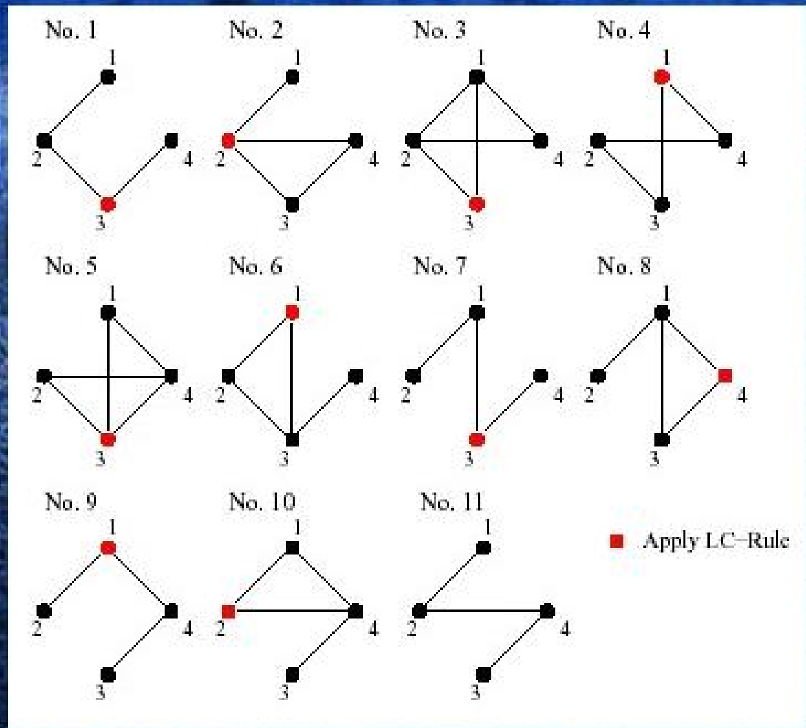
A nice example of local complementation and graph isomorphism

Introduction to Graph States and their properties

***Entanglement** in graph states is not always what it seems...*



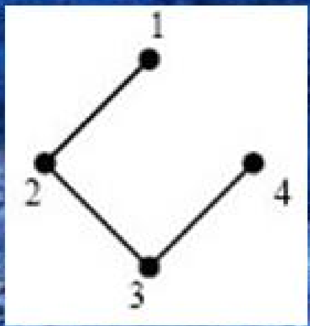
$$U_a(G) = \left(-i\sigma_x^{(a)}\right)^{1/2} \prod_{b \in N_a} \left(i\sigma_z^{(b)}\right)^{1/2}$$

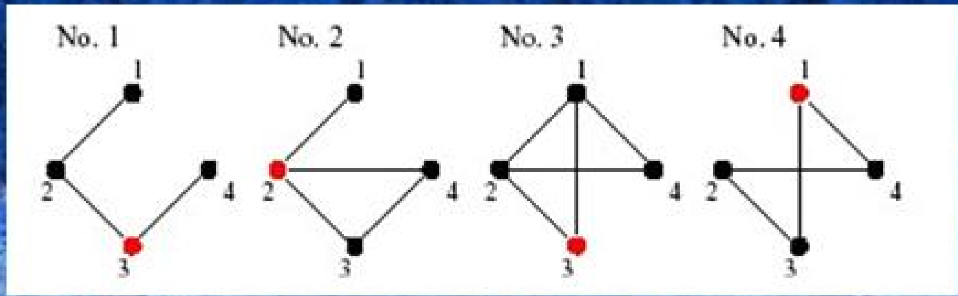


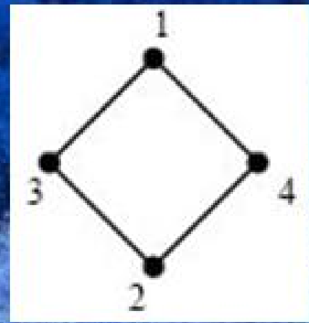
-Hein et al., quant-ph/0602096
-Hein, Eisert and Briegel, PRA (2004)

Introduction to Graph States and their properties

Entanglement in graph states is not always what it seems...







$$\tilde{Z}_1\tilde{Z}_1 \otimes \tilde{Z}_2\tilde{X}_2\tilde{Z}_2 \otimes \tilde{X}_3\tilde{Z}_3\tilde{X}_3 \otimes \tilde{Z}_4\tilde{Z}_4$$

$$U_a(G) = \left(-i\sigma_x^{(a)}\right)^{1/2} \prod_{b\in N_a} \left(i\sigma_z^{(b)}\right)^{1/2}$$

$\tilde{X}$ $\tilde{Z}$

$$i\sigma_z \otimes i\sigma_z\sigma_x H \otimes i\sigma_z\sigma_x H \otimes i\sigma_z$$

Introduction to Graph States and their properties

Producing a box cluster in an experiment

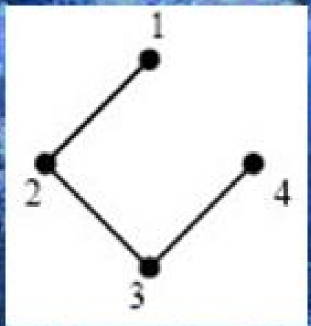
P. Walther et al., Nature (2005)

R. Prevedel et al., Nature (2006)

+other groups...

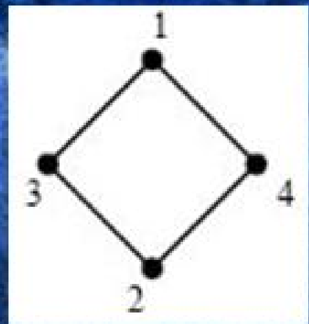
$$|\phi_{exp}\rangle = (|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle)_{1234}$$

$$H \otimes \mathbb{1} \otimes \mathbb{1} \otimes H$$



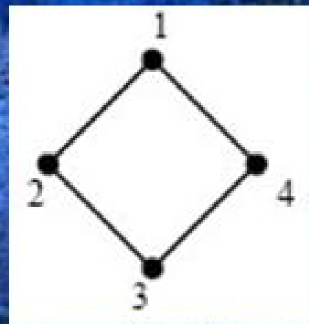
$|\phi_{lin}\rangle$

$$i\sigma_z \otimes i\sigma_z\sigma_x H \otimes i\sigma_z\sigma_x H \otimes i\sigma_z$$



$|\phi_{\overline{box}}\rangle$

Relabel vertices 2 and 3



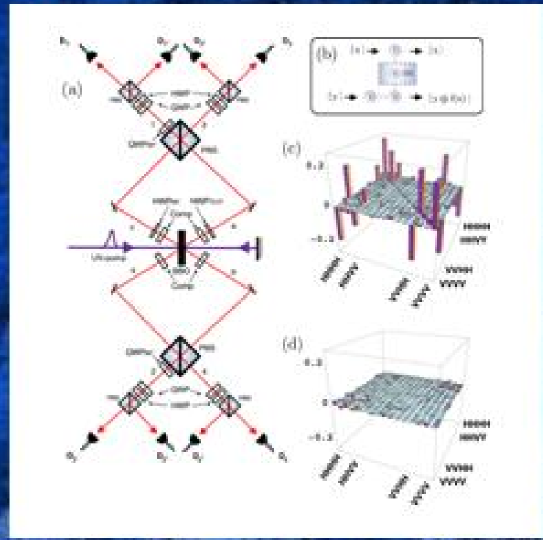
$|\phi_{box}\rangle$

$$i\sigma_z H \otimes i\sigma_z\sigma_x H \otimes i\sigma_z\sigma_x H \otimes i\sigma_z H \; |\phi_{exp}\rangle = |\phi_{box}\rangle$$

(+*Relabel* vertices 2 and 3)

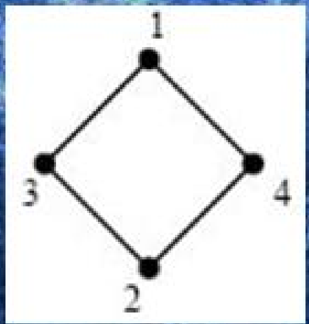
Introduction to Graph States and their properties

Producing a box cluster in an experiment



$$|\phi_{exp}\rangle = (|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle)_{1234}$$

$$i\sigma_z H \otimes i\sigma_z\sigma_x H \otimes i\sigma_z\sigma_x H \otimes i\sigma_z H \quad |\phi_{exp}\rangle = |\phi_{\overline{box}}\rangle$$



$|\phi_{\overline{box}}\rangle$

$$K_G^{(a)}|G\rangle = |G\rangle$$

$$K_G^{(a)} = \sigma_x^{(a)} \prod_{b \in N_a} \sigma_z^{(b)} .$$

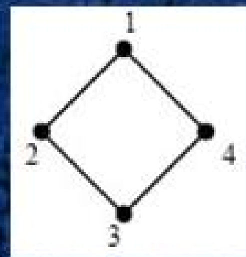
$$K^2 = \sigma_x^2 \otimes \sigma_z^3 \otimes \sigma_z^4$$

$$K^3 = \sigma_x^3 \otimes \sigma_z^1 \otimes \sigma_z^2$$

$$K^3 K^2 = \sigma_z^1 \otimes (\sigma_z\sigma_x)^2 \otimes (\sigma_z\sigma_x)^3 \otimes \sigma_z^4$$

Therefore $H \otimes H \otimes H \otimes H \quad |\phi_{exp}\rangle = |\phi_{\overline{box}}\rangle$

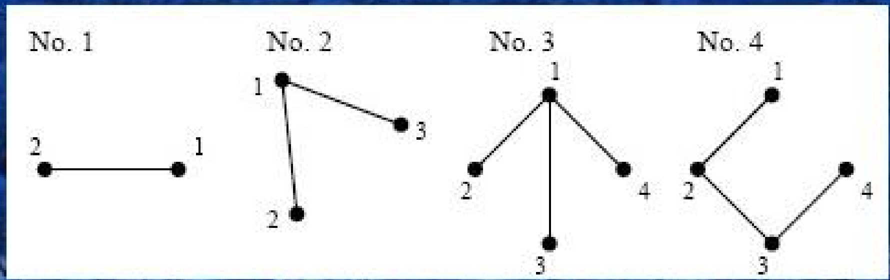
...and finally $H \otimes H \otimes H \otimes H \quad |\phi_{exp}\rangle = |\phi_{box}\rangle$



(+*Relabel* vertices 2 and 3)

Introduction to Graph States and their properties

General classes of graph states: n=2, 3 and 4

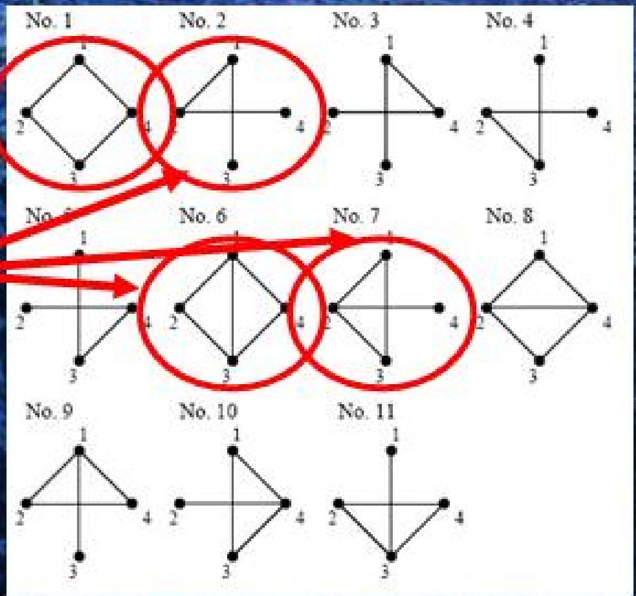


List of connected graphs up to 4 vertices that are not equivalent under LU transformations and graph isomorphisms

|LU class|= 1 2 2 4

Number of non-isomorphic but LU-equivalent graphs

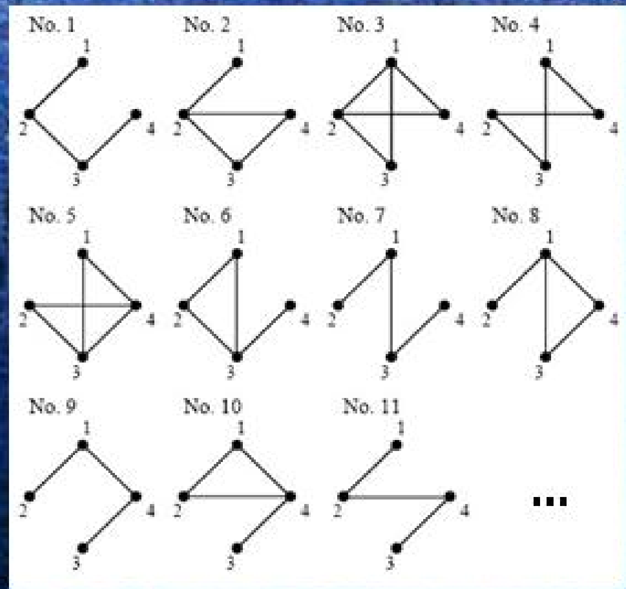
e.g. All other graphs are isomorphic to these 4



Complete LU equivalence class of graph No. 1 (Orbit)

Not LU equivalent

But equivalent under LU and isomorphism

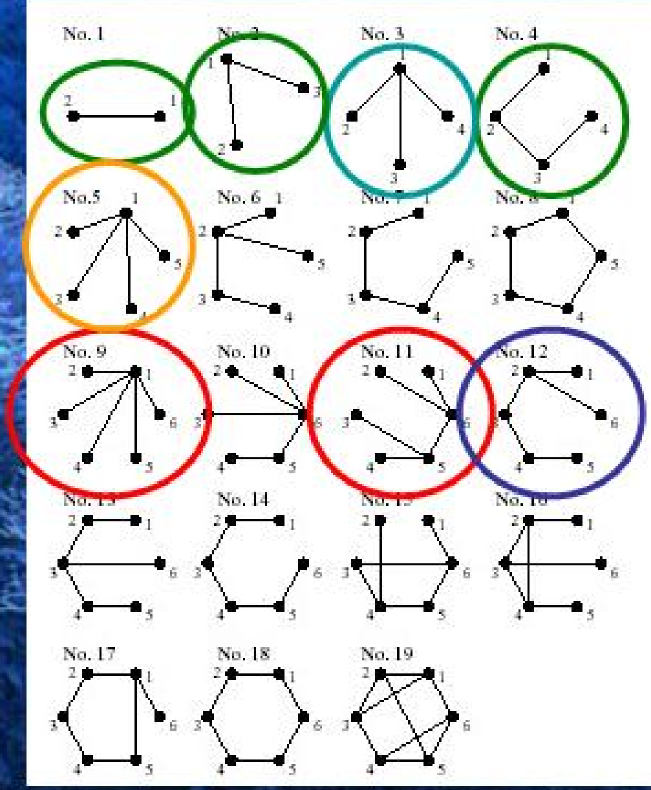


another LU equivalence class

Introduction to Graph States and their properties

General classes of graph states up to n=6:

-Hein et al., quant-ph/0602096



No.	|LUclass|	$\|V\|$	$\|E\|$	SR_{max}	PP	RI_3	RI_2	$2-col$
1	1	2	1	1	1			yes
2	2	3	2	1	1			yes
3	2	4	3	1	1		(0,3)	yes
4	4	4	3	2	2		(2,1)	yes
5	2	5	4	1	1		(0,10)	yes
6	6	5	4	2	2		(6,4)	yes
7	10	5	4	2	2		(8,2)	yes
8	3	5	5	2	3		(10,0)	no
9	2	6	5	1	1	(0,0,10)	(0,15)	yes
10	6	6	5	2	2	(0,6,4)	(8,7)	yes
11	4	6	5	2	2	(0,9,1)	(8,7)	yes
12	16	6	5	2	2	(0,9,1)	(11,4)	yes
13	10	6	5	3	3	(4,4,2)	(12,3)	yes
14	25	6	5	3	3	(4,5,1)	(13,2)	yes
15	5	6	6	2	2	(0,10,0)	(12,3)	yes
16	5	6	6	3	3	(4,6,0)	(12,3)	yes
17	21	6	6	3	3	(4,6,0)	(14,1)	yes
18	16	6	6	3	3	(6,4,0)	(15,0)	yes
19	2	6	9	3	4	(10,0,0)	(15,0)	no

Linear optical Experimental generation:

Many groups

Not so many groups

1 group – Zhao et al., Nature (2004)

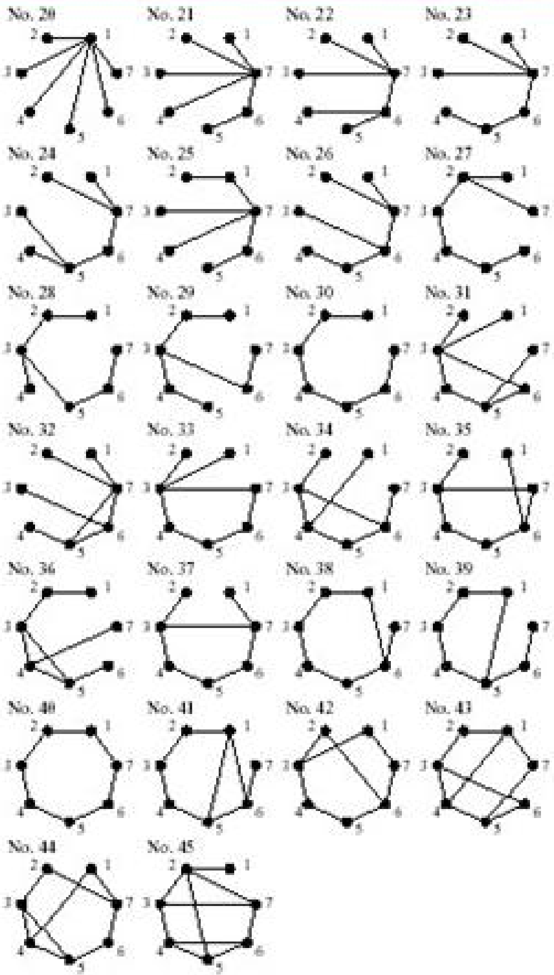
1 group – Lu et al., Nature Physics (2007)

1 group - Lu et al., arXiv:0710.0278 (2007)

Introduction to Graph States and their properties

General classes of graph states, n=7

-Hein et al., quant-ph/0602096



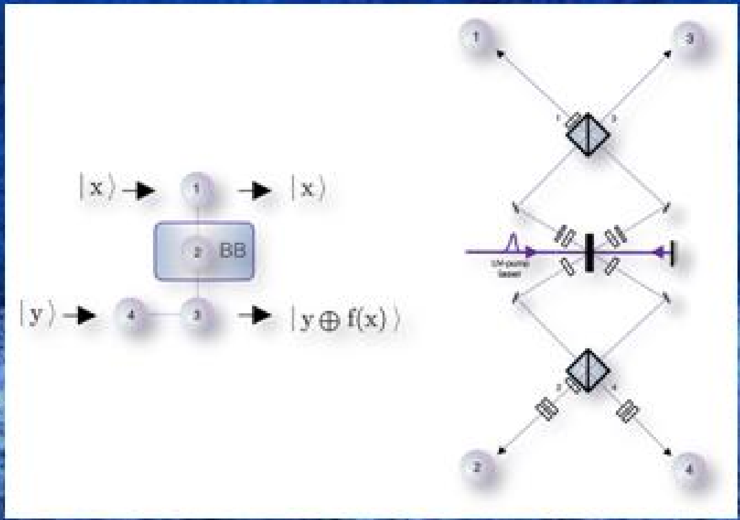
No.	$\lvert\mathrm{LUclass}\rvert$	$\lvert V\rvert$	$\lvert E\rvert$	$\mathrm{SR}_{\max}$	PP	RI_3	RI_2	$2-col$
20	2	7	6	1	1	(0,0,35)	(0,21)	yes
21	6	7	6	2	2	(0,20,15)	(10,11)	yes
22	6	7	6	2	2	(0,30,5)	(12,9)	yes
23	16	7	6	2	2	(0,30,5)	(14,7)	yes
24	10	7	6	2	2	(0,33,2)	(15,6)	yes
25	10	7	6	3	3	(12,16,7)	(16,5)	yes
26	16	7	6	3	3	(12,20,3)	(16,5)	yes
27	44	7	6	3	3	(12,21,2)	(17,4)	yes
28	44	7	6	3	3	(16,16,3)	(18,3)	yes
29	14	7	6	3	3	(20,12,3)	(18,3)	yes
30	66	7	6	3	3	(20,13,2)	(19,2)	yes
31	10	7	7	2	2	(0,34,1)	(16,5)	yes
32	10	7	7	3	3	(12,22,1)	(16,5)	no
33	21	7	7	3	3	(12,22,1)	(18,3)	no
34	26	7	7	3	3	(16,18,1)	(18,3)	yes
35	36	7	7	3	3	(16,19,0)	(19,2)	no
36	28	7	7	3	3	(20,14,1)	(18,3)	no
37	72	7	7	3	3	(20,15,0)	(19,2)	no
38	114	7	7	3	3	(22,13,0)	(20,1)	yes
39	56	7	7	3	4	(24,10,1)	(20,1)	no
40	92	7	7	3	4	(28,7,0)	(21,0)	no
41	57	7	8	3	4	(26,9,0)	(20,1)	no
42	33	7	8	3	4	(28,7,0)	(21,0)	no
43	9	7	9	3	3	(28,7,0)	(21,0)	yes
44	46	7	9	3	4	(32,3,0)	(21,0)	no
45	9	7	10	3	4	(30,5,0)	(20,1)	no

and more!...

Q: Why should we generate them at all?

Introduction to Graph States and their properties

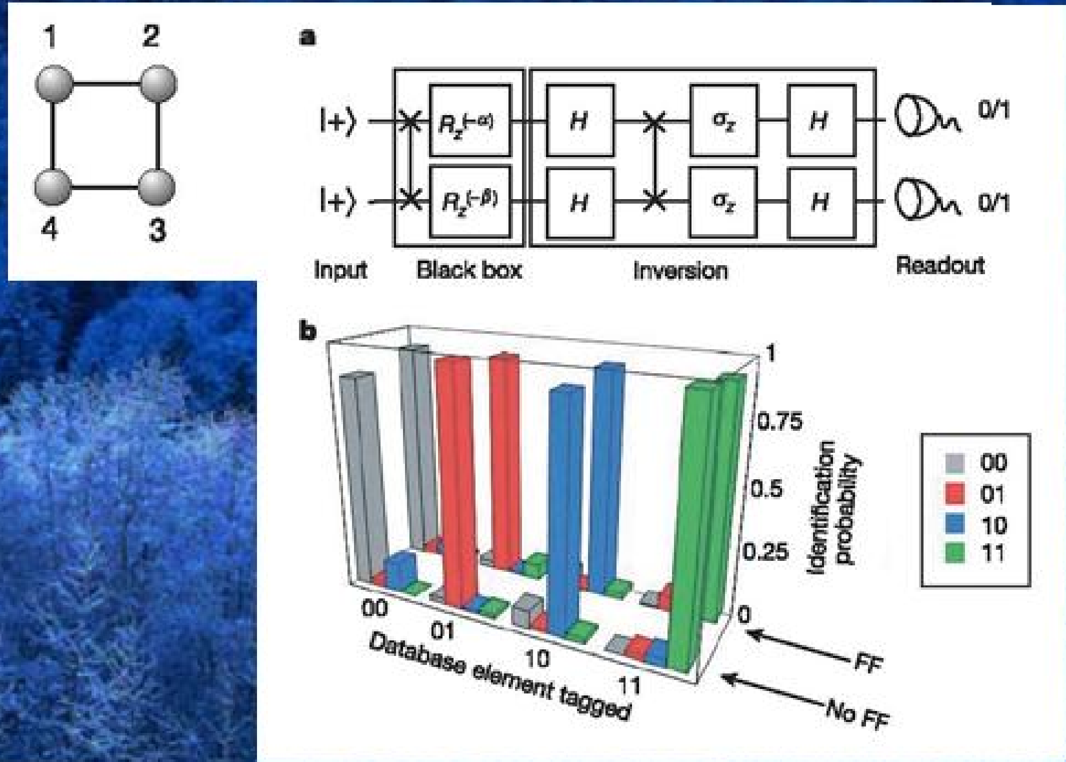
Protocols!



Deutsch's Algorithm:

M. S. Tame et al., PRL (2007)

Grover's Algorithm:



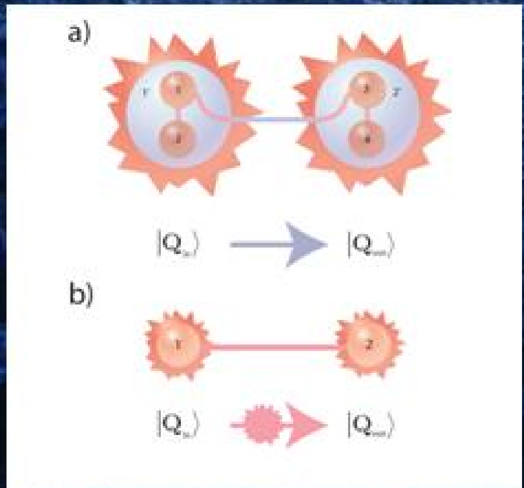
P. Walther et al., Nature (2005)

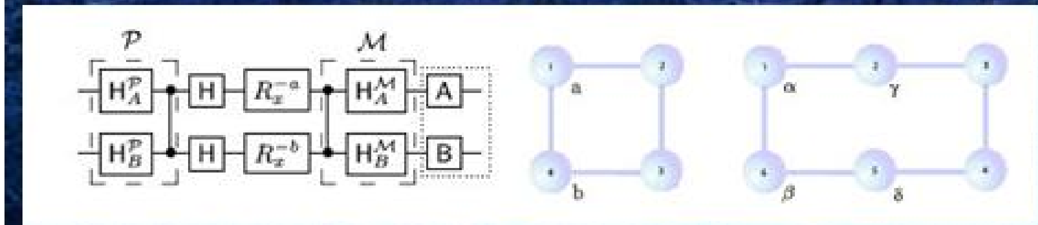
R. Prevedel et al., Nature (2006)

Decoherence-free subspace processing:

M. S. Tame et al., NJP (2007)

R. Prevedel et al., PRL (2007)





Quantum Games:

M. Paternostro, M. S. Tame, M. S. Kim, NJP (2005)

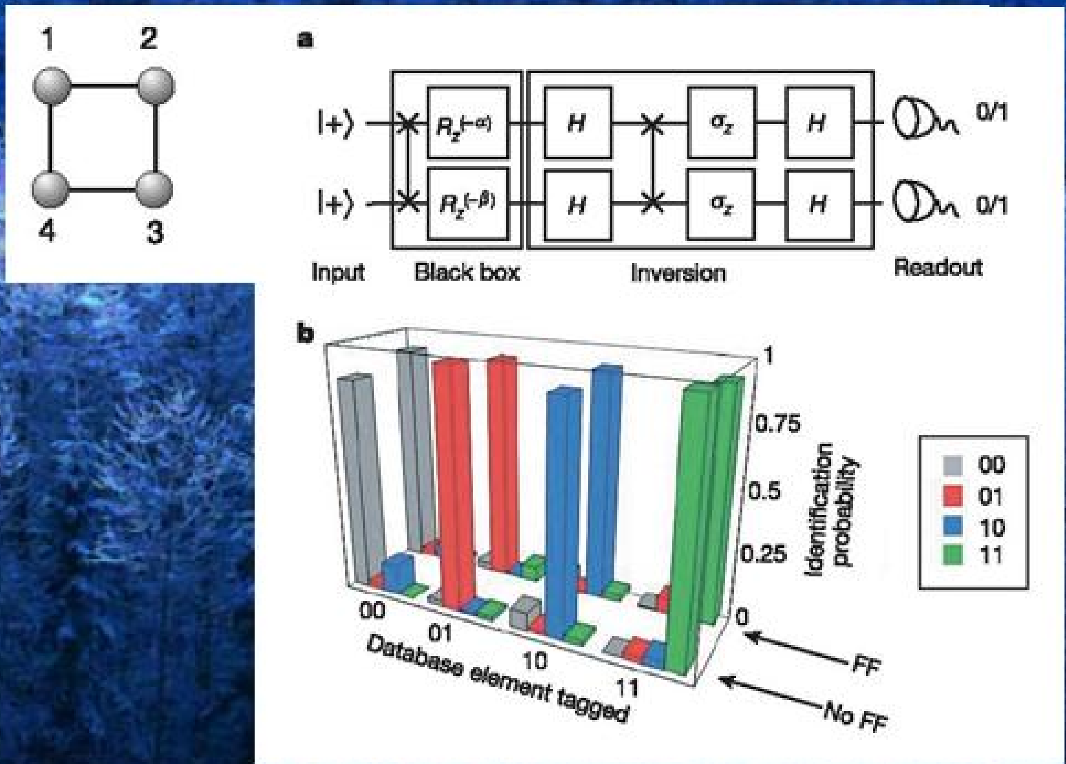
R. Prevedel et al., NJP (2007)

Protocols.

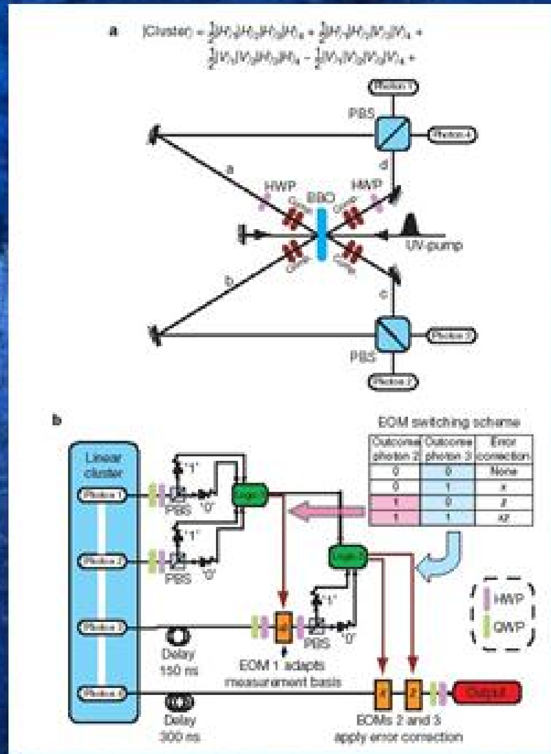
Grover's Algorithm:

L .K. Grover, Phys. Rev. Lett. 79, 325 (1997).

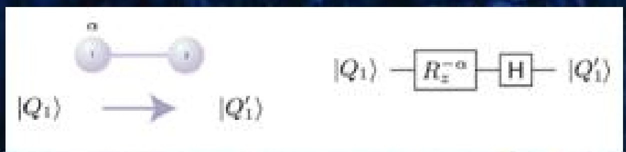
and active feed-forward!

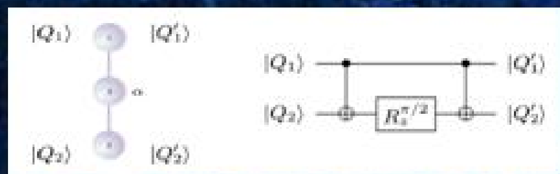


P. Walther et al., Nature (2005)



R. Prevedel et al., Nature (2006)

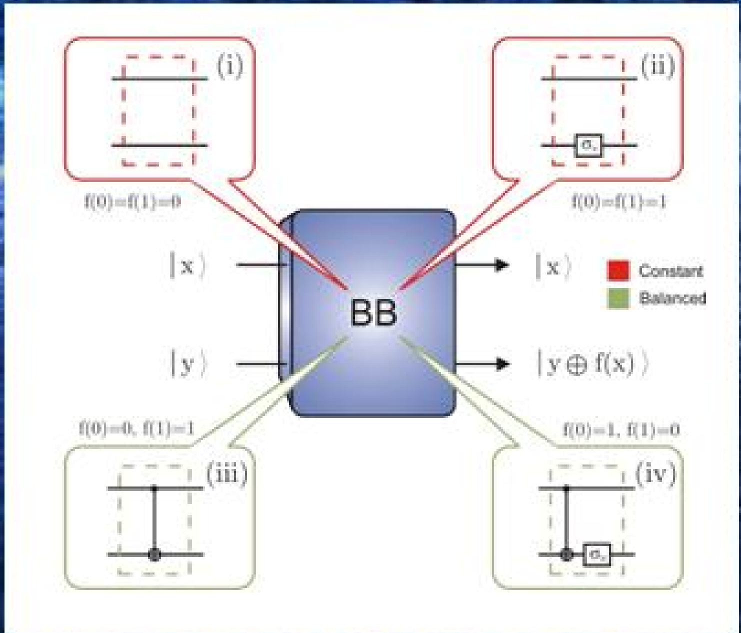


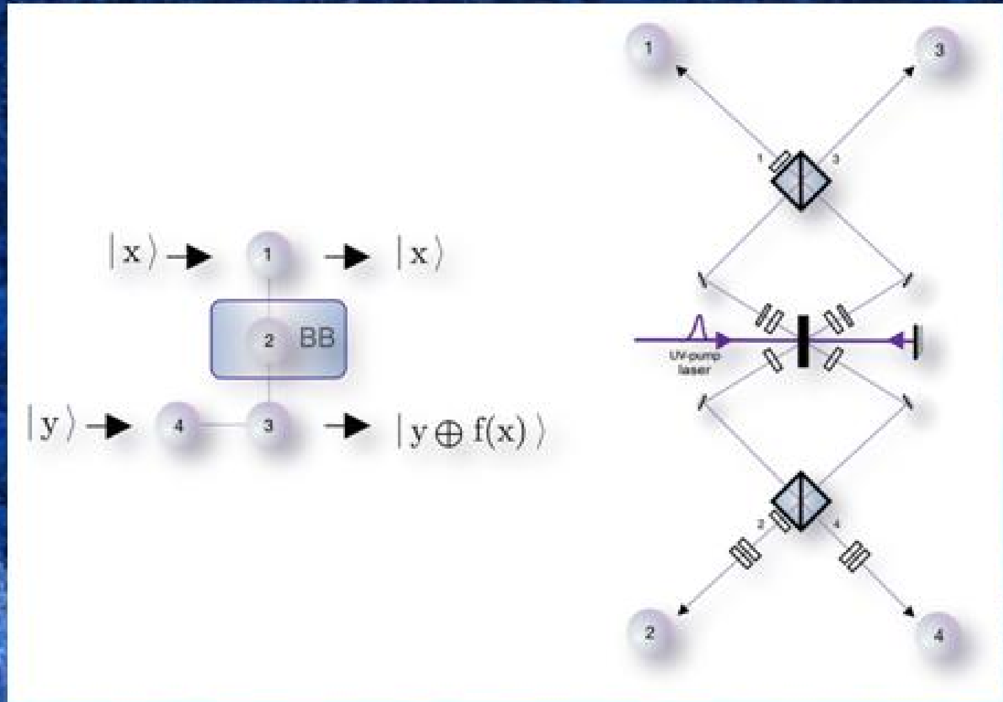


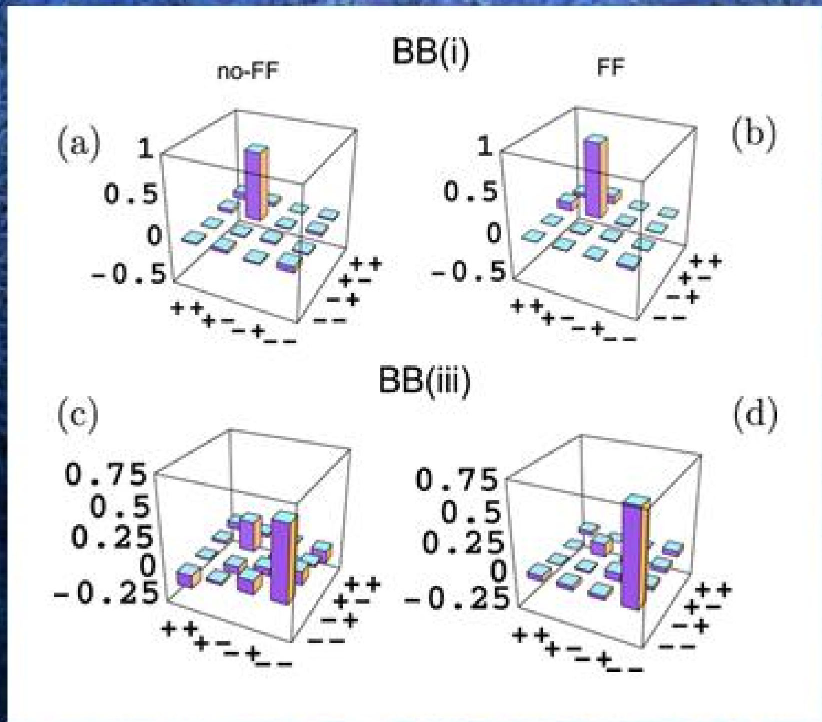
Protocols.

Deutsch's Algorithm:

M. S. Tame et al., PRL (2007)





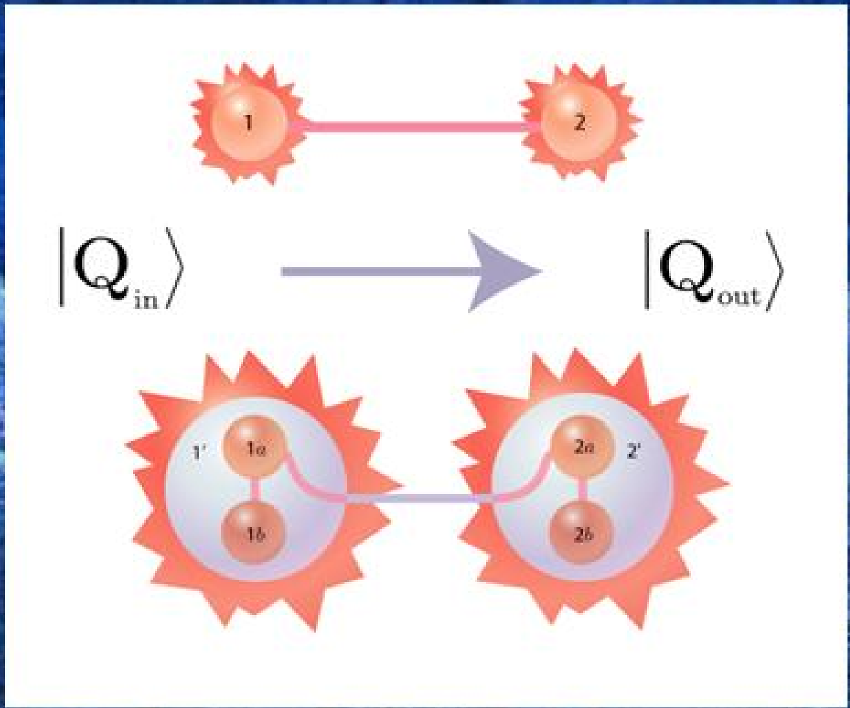


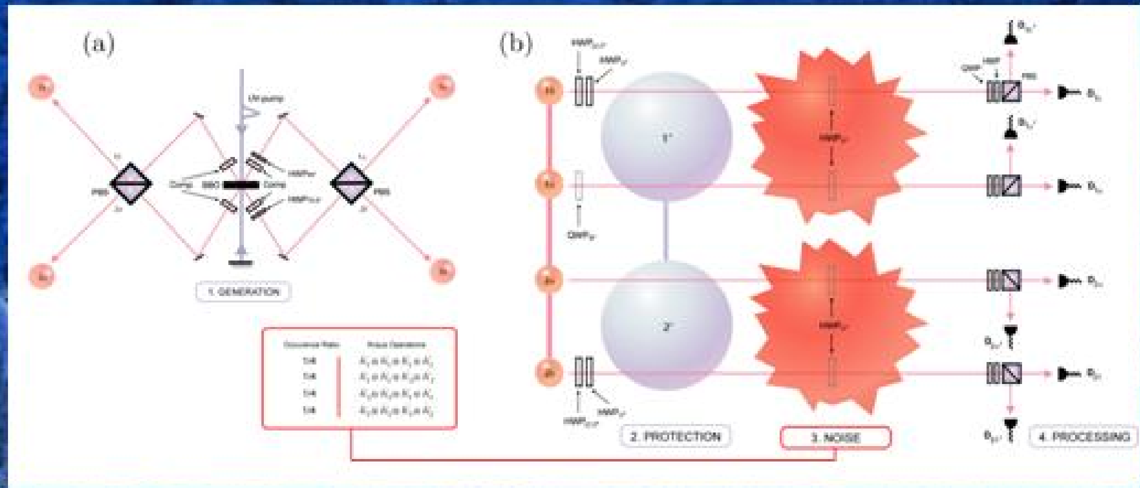
$$|x\rangle\,|y\rangle = |+\rangle\,|-\rangle$$

$$(1/\sqrt{2})[(-1)^{f(0)}\,|0\rangle + (-1)^{f(1)}\,|1\rangle]\,|-\rangle$$

Protocols.

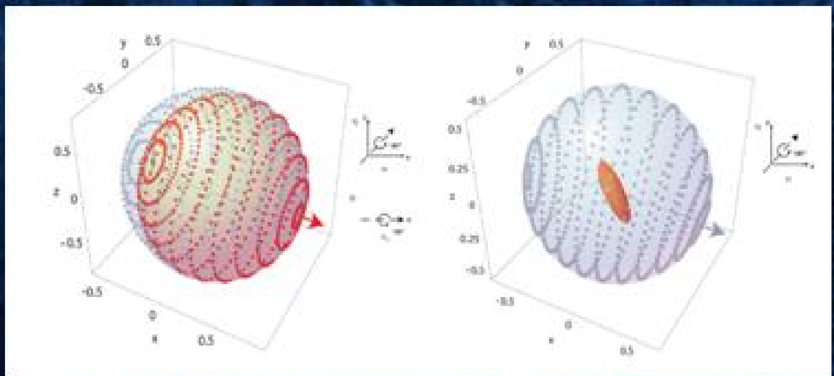
Linear optical setup





Information transfer protocol: <u>4 physical qubits</u>

R. Prevedel, M. S. Tame, A. Stefanov,
M. Paternostro, M. S. Kim and A. Zeilinger
PRL (2007)



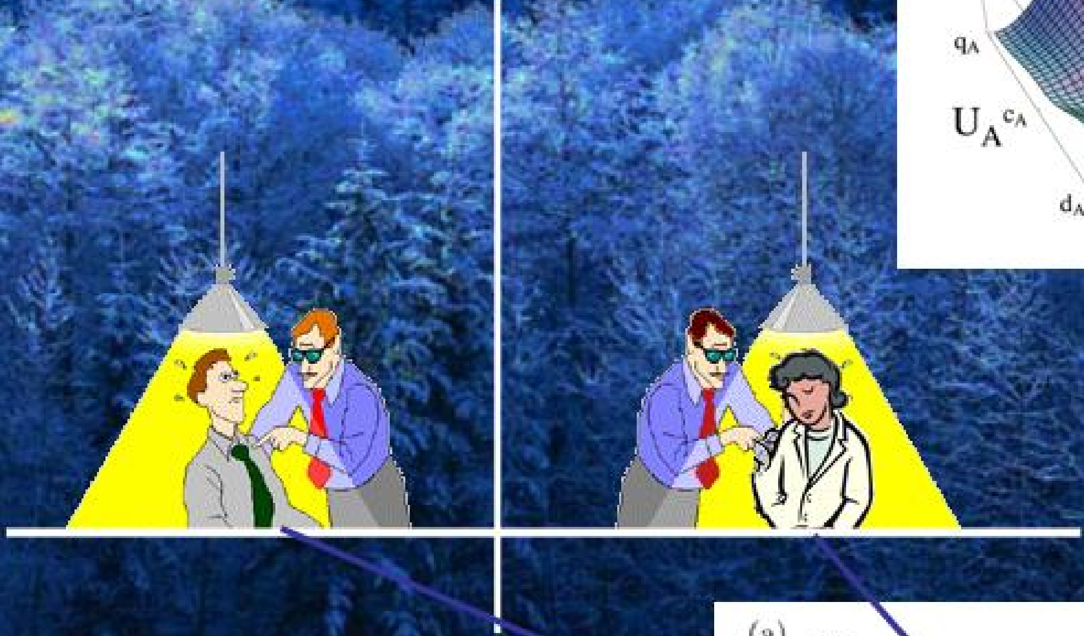
See also:
Kwiat et al., Science 290, 498-501 (2000)
for single qubit DFS encoding.

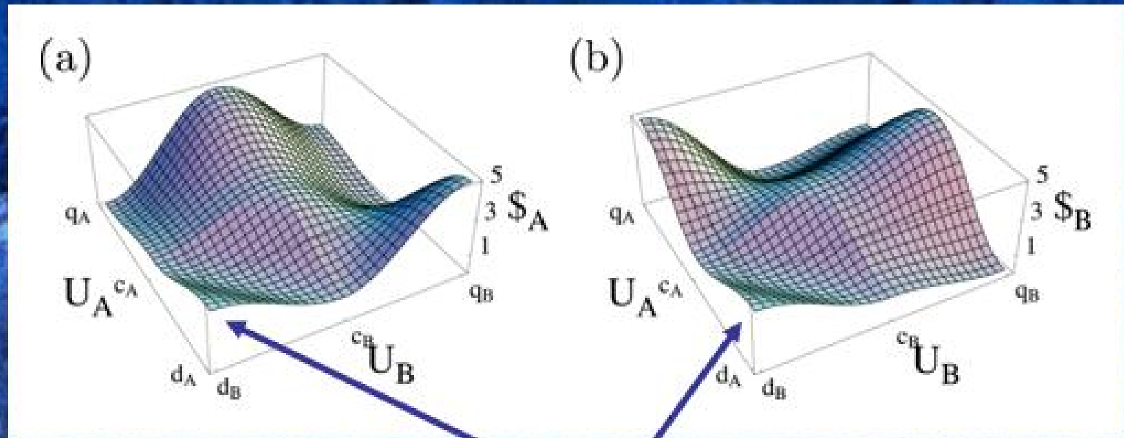
Protocols.

		BOB: cooperate	BOB: defect
ALICE	cooperate	(3,3)	(0,5)
	defect	(5,0)	(1,1)

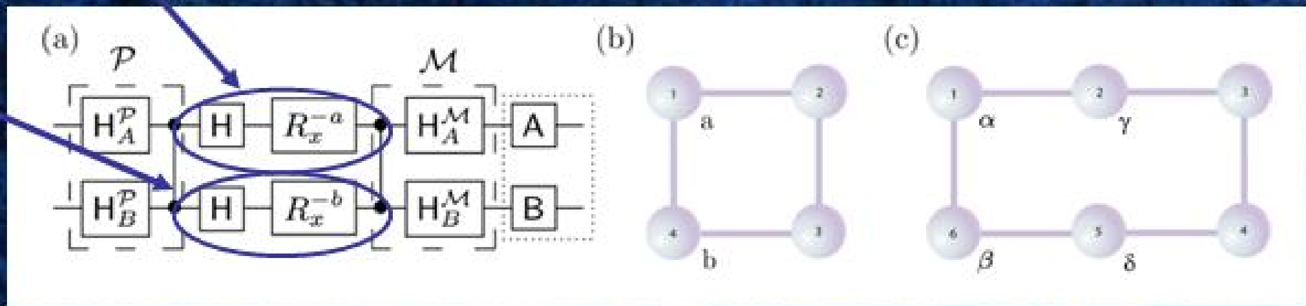
Quantum Games:

J. Eisert, M. Wilkens, M. Lewenstein, PRL (1999)
M. Paternostro, M. S. Tame, M. S. Kim, NJP (2005)
R. Prevedel et al., NJP (2007)





Pareto Optimal / Nash Equilibrium
(3,3) Payoff



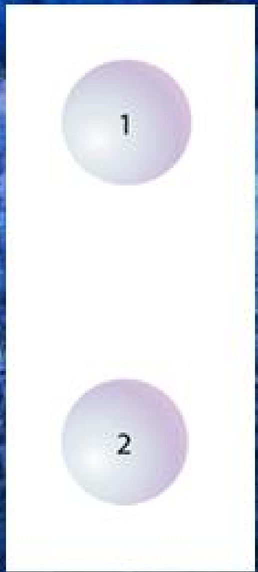
Weighted Graph States

A simple <u>weighted</u> graph state

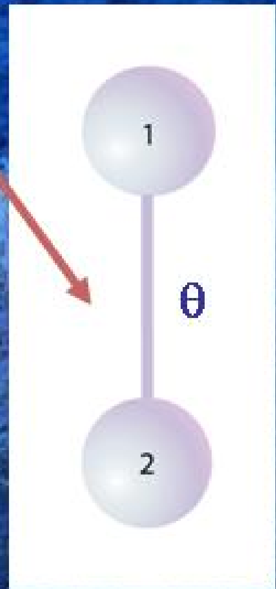
2) Application of CZ^{θ}

1) Preparation of $|+\rangle$

$CZ^{\theta} = diag(1, 1, 1, e^{i\theta_{jk}})$



CZ^{θ}: |0> |0> --> |0> |0>
|0> |1> --> |0> |1>
|1> |0> --> |1> |0>
|1> |1> --> exp[iθ] |1> |1>



$CZ^{\theta} = diag(1, 1, 1, e^{i\theta_{jk}})$

$weights\ \theta_{jk} \in [0, \pi]$

$\theta_{jk} = \pi$ $(\theta_{jk} = 0)$

controlled phase (identity)

| ψ > = |+> |+>

$|\pm\rangle_j = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)_j$

| G > = (|0> |+> + |1> |->)

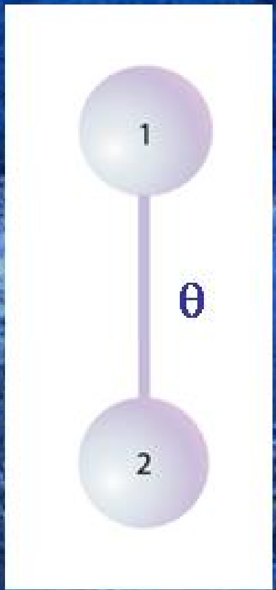
| G_w > = (|0> |+> + |1> |θ_+>)

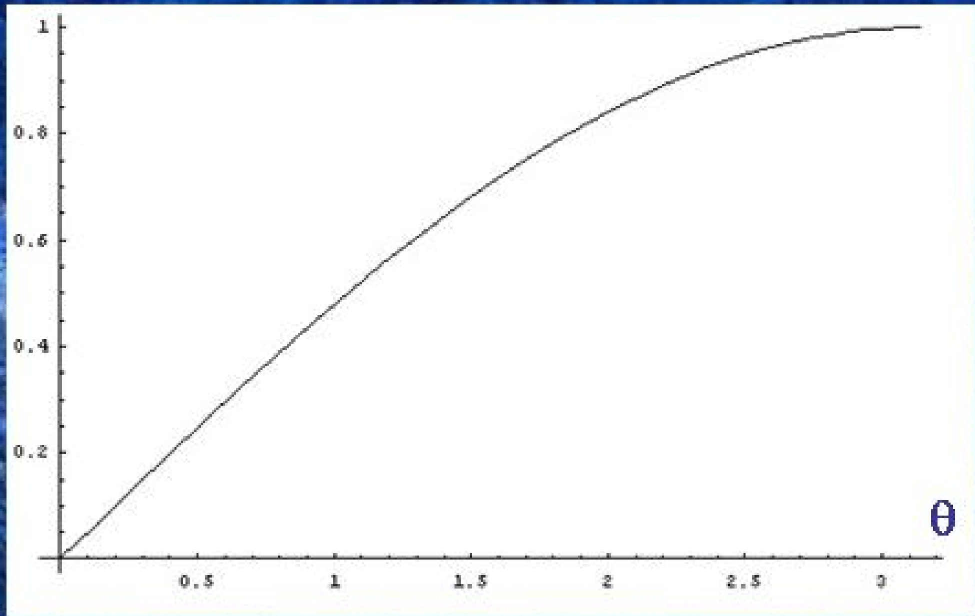
$|\alpha_\pm\rangle_j = \frac{1}{\sqrt{2}}(|0\rangle \pm e^{i\alpha}|1\rangle)_j$

Weighted Graph States

A simple <u>weighted</u> graph state

Concurrence





$C = \sin\theta/2$

$CZ^{\theta} = diag(1,1,1,e^{i\theta_{jk}})$

$weights\ \theta_{jk} \in [0,\pi]$

$\theta_{jk} = \pi$ $(\theta_{jk} = 0)$

controlled phase (identity)

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\},$$

where the λ_i's are the square roots of the eigenvalues of the non-Hermitian matrix $\tau = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ in decreasing order. Here ρ^* is the complex conjugate of ρ in the computational basis and each λ_i is a non-negative real number.

Thanks for listening

References:

- **Raussendorf, Browne and Briegel,** PRA 68, 022312 (2003).
- **Hein et al.,** *Proceedings of the International School of Physics "Enrico Fermi" on "Quantum Computers, Algorithms and Chaos",* Varenna, Italy, July, 2005; also at quant-ph/0602096
- **Hein, Eisert and Briegel,** PRA (2004)
- **Browne and Rudolph**, PRL (2005)
- **P. Walther** et al., Nature (2005)
- **R. Prevedel** et al., Nature (2006)
- **M. Paternostro**, M. S. Tame, M. S. Kim, NJP (2005)
- **R. Prevedel** et al., NJP (2007)
- **M. S. Tame** et al., NJP (2007)
- **R. Prevedel** et al., PRL (2007)
- **M. S. Tame** et al., PRL (2007)