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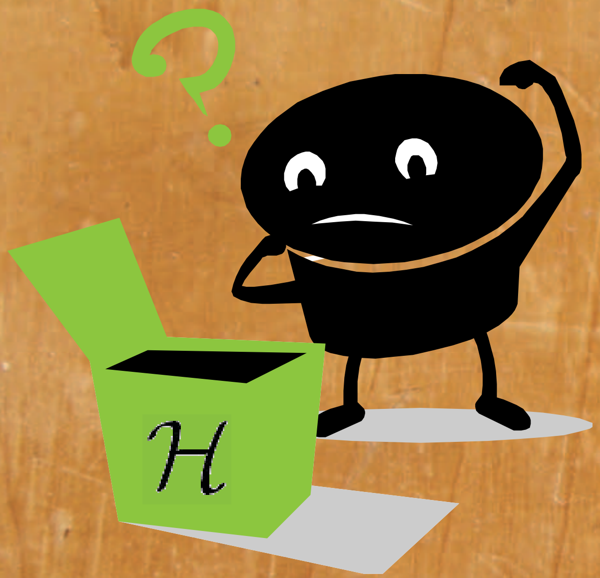
Dr. Mark Tame
Queen's University Belfast
Photonics Workshop Bristol, 18th Sept 2009

EXPERIMENTAL REALIZATION OF MANY-PHOTON SYMMETRIC STATES FOR MULTIPARTY QUANTUM NETWORKING

INTRODUCTION

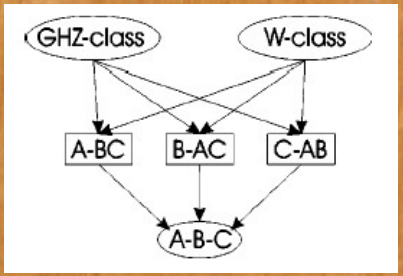


Hilbert space:

$$\mathcal{H}^{(N)} = \underbrace{\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2}_{N},$$

$$2(2^N - 1)$$

SLOCC classification:



3 qubits

W. Dur, G. Vidal and J. I. Cirac, Phys. Rev. A 62, 062314 (2000).
A. Acin et al., Phys. Rev. Lett. 87, 040401 (2001).

F. Verstraete *et al.*, Phys. Rev. A 65, 052112 (2002).
L. Chen and Y. Chen, Phys. Rev. A *74, 062310 (2006)*
L. Lamata et al., Phys Rev. A 75, 022318 (2007).

TABLE I. Genuine entanglement classes for four qubits.

Class ($\mathfrak{W}$)	Canonical states	Name	Proposed notation
span{000,000}	$\|0000\rangle+\|1111\rangle$	GHZ	$\mathfrak{W}_{000,000}$
span{000, $0_k\Psi$}	$\|0000\rangle+\|1100\rangle+\|1111\rangle$		$\mathfrak{W}_{000,0_k\Psi}$
	$\|0000\rangle+\|1101\rangle+\|1110\rangle$		
span{000, GHZ}	$\|0\phi\varphi\psi\rangle+\|1000\rangle+\|1111\rangle$		$\mathfrak{W}_{000,\mathrm{GHZ}}$
span{000, W}	$\|1000\rangle+\|0100\rangle+\|0010\rangle+\|0001\rangle$	W	$\mathfrak{W}_{000,W}$
span{$0_k\Psi$, $0_k\Psi$}	$\|0000\rangle+\|1100\rangle+\lambda_1\|0011\rangle+\lambda_2\|1111\rangle$	Φ_4	$\mathfrak{W}_{0_k\Psi,0_k\Psi}$
	$\|0000\rangle+\|1100\rangle+\lambda_1\|0001\rangle+\lambda_1\|0010\rangle+\lambda_2\|1101\rangle+\lambda_2\|1110\rangle$		
span{$0_i\Psi$, $0_j\Psi$}	$\|0\phi00\rangle+\|0\phi1\psi\rangle+\|1000\rangle+\|1101\rangle$		$\mathfrak{W}_{0_i\Psi,0_j\Psi}$
	$\|0\phi0\psi\rangle+\|0\phi10\rangle+\|1000\rangle+\|1101\rangle$		
span{$0_k\Psi$, GHZ}	$\|0\phi\Psi\rangle+\|1000\rangle+\|1111\rangle$		$\mathfrak{W}_{0_k\Psi,\mathrm{GHZ}}$
span{GHZ, W}	$\|0001\rangle+\|0010\rangle+\|0100\rangle+\|1\phi\varphi\psi\rangle+\|1\bar{\phi}\bar{\varphi}\bar{\psi}\rangle$		$\mathfrak{W}_{\mathrm{GHZ},W}$

4 qubits

...and beyond!

INTRODUCTION

Cluster and graph states:

$$K^{(a)}|\phi_{\{\kappa\}}\rangle_C = (-1)^{\kappa_a}|\phi_{\{\kappa\}}\rangle_C,$$

$$K^{(a)} = \sigma_x^{(a)} \bigotimes_{b \in \mathrm{nghb}(a)} \sigma_z^{(b)}.$$

GHZ states: $1/\sqrt{2}(|00\cdots0\rangle + |11\cdots1\rangle)$

$$|\mathrm{GHZ}\rangle = (1/\sqrt{2})(|000\rangle + |111\rangle)$$

same SLOCC class

$$|\phi\rangle_{C_2} = \frac{1}{\sqrt{2}}(|0\rangle_1|+\rangle_2 + |1\rangle_1|-\rangle_2),$$

$$|\phi\rangle_{C_3} = \frac{1}{\sqrt{2}}(|+\rangle_1|0\rangle_2|+\rangle_3 + |-\rangle_1|1\rangle_2|-\rangle_3),$$

$$|\phi\rangle_{C_4} = \frac{1}{2}|+\rangle_1|0\rangle_2|+\rangle_3|0\rangle_4 + \frac{1}{2}|+\rangle_1|0\rangle_2|-\rangle_3|1\rangle_4 + \frac{1}{2}|-\rangle_1|1\rangle_2|-\rangle_3|0\rangle_4 + \frac{1}{2}|-\rangle_1|1\rangle_2|+\rangle_3|1\rangle_4,$$

D.M. Greenberger *et al.*, Am. J. Phys. 58, 1131 (1990).

W states: $|W_N\rangle \equiv (1/\sqrt{N})|N-1,1\rangle,$

$$|W\rangle = (1/\sqrt{3})(|001\rangle + |010\rangle + |100\rangle),$$

$$|W_4\rangle = (1/\sqrt{4})(|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle).$$

R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. 86, 5188 (2001).
R. Raussendorf, D. E. Browne, and H. J. Briegel, Phys. Rev. A 68, 022312 (200
H. J. Briegel *et al.*, Nature Phys. 5, 19 (2009).

W. Dur, G. Vidal and J. I. Cirac, Phys. Rev. A 62, 062314 (2000).
A. Acin et al., Phys. Rev. Lett. 87, 040401 (2001).

Singlet/decoherence-free states
Spin-squeezed states
Comb states
NOON states ...and more

INTRODUCTION

Dicke states:

$$|D_n^{(k)}\rangle = \frac{1}{\sqrt{C_n^k}} \sum_l \hat{P}_l |0_0 \ldots 0_k 1_{k+1} \ldots 1_n\rangle,$$

Dicke state of n qubits and k excitations $|D_n^{(k)}\rangle$

$\hat{J}^2 = \sum_{k=x,y,z} \hat{J}_k^2$ $\quad \hat{J}_z$

$\hat{J}_k = (1/2) \sum_{j=1}^n \hat{\sigma}_k^j$

R.H. Dicke, Phys. Rev. 93, 99 (1954)

$|D_N^{(N)}\rangle = |0000\ldots\rangle$ ← I α N

$|D_N^{(N-1)}\rangle = (|1000\ldots\rangle + |0100\ldots\rangle + |0010\ldots\rangle + \ldots)$

$|D_N^{(N-1)}\rangle, |D_N^{(N-2)}\rangle, \ldots, |D_N^{(N/2)}\rangle, \ldots, |D_N^{(0)}\rangle$

I α N^2

Superradiance

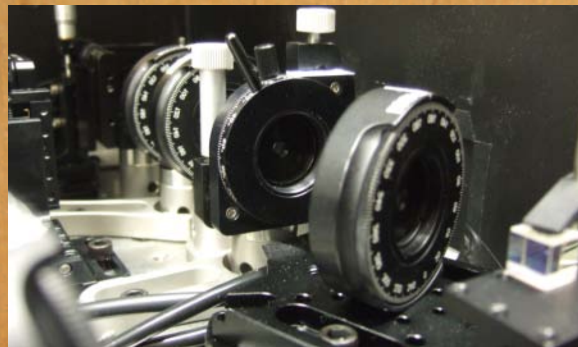
T. Brandes, Phys. Rep. 408, 315 (2005).
N. Lambert, C. Emary and T. Brandes, Phys. Rev. Lett. 92, 073602 (2004).
J. Vidal, S. Dusuel, and T. Barthel, J. Stat. Mech. P01015 (2007).

V.Buzek, M. Orszag, and M. Rosko, Phys. Rev. Lett. 94, 163601 (2005).
C. E. Lopez, J. C. Retamal, and E. Solano, Phys. Rev. A 76, 033413 (2007). C. Thiel *et al.*, Phys. Rev. Lett. 99, 193602 (2007).

I. Generation of photonic Dicke states

II. Characterisation techniques

III. Quantum information applications



Experimental realisation of Dicke states of up to six-qubits for multiparty quantum networking

R. Prevedel[1], G. Cronenberg[1], M. S. Tame[2,*], M. Paternostro[2], P. Walther[1,3], M. S. Kim[2] and A. Zeilinger[1,3]

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[2]School of Mathematics and Physics, The Queen's University, Belfast, BT7 1NN, UK
[3]Institute for Quantum Optics and Quantum Information (IQOQI), Austrian Academy of Sciences, Boltzmanngasse 3, A-1090 Vienna, Austria

Physical Review Letters 103, 020503 (2009)

Characterizing multipartite symmetric Dicke states under the effects of noise

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School of Mathematics and Physics, The Queen's University, Belfast, BT7 1NN, UK

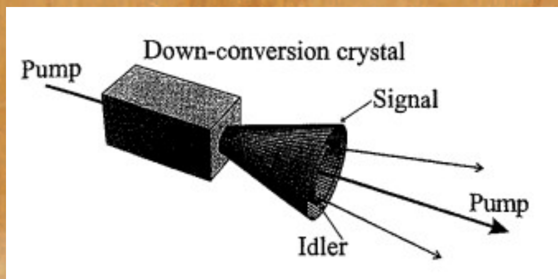
New Journal of Physics 11, 073039 (2009)

W. Wieczorek *et al.*, Phys. Rev. Lett. 103, 020504 (2009)
G. Toth et al., New. J. Phys. 11, 083002 (2009)

I. GENERATION

I. GENERATION

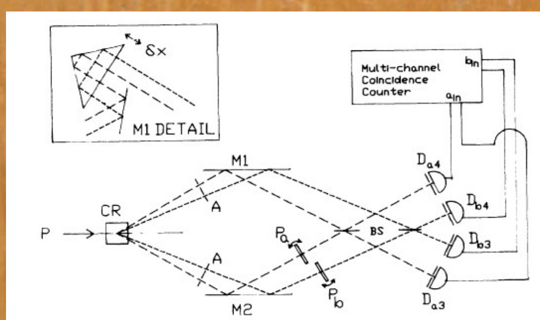
PDC-I: single cone



Gerry and Knight, Introductory Quantum Optics

$$\hat{H}_\mathrm{I} = \hbar\eta\, \hat{a}_\mathrm{s}^\dagger \hat{a}_\mathrm{i}^\dagger + H.c.,$$

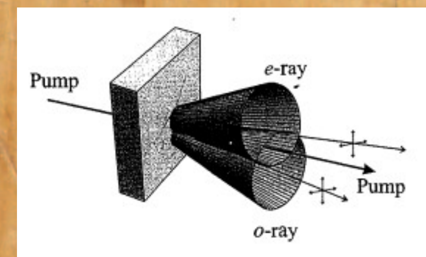
$$\eta \propto \chi^{(2)}\mathcal{E}_\mathrm{p}$$



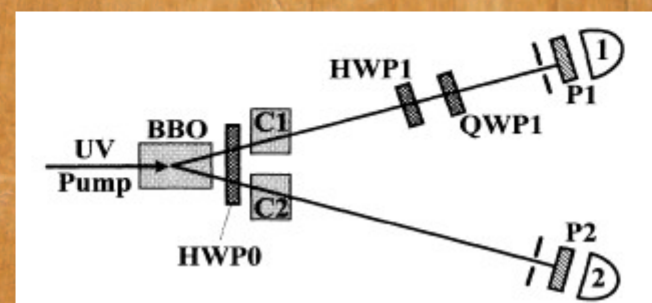
Shih and Alley, Phys. Rev. Lett. 61, 2921 (1988)
Ou and Mandel, Phys. Rev. Lett. 61, 50 (1988)
Rarity and Tapster, Phys. Rev. Lett. 64, 2495 (1990)

and many more…

PDC-II: two cones



$$\hat{H}_\mathrm{I} = \hbar\eta\left(\hat{a}_\mathrm{Vs}^\dagger \hat{a}_\mathrm{Hi}^\dagger + \hat{a}_\mathrm{Hs}^\dagger \hat{a}_\mathrm{Vi}^\dagger\right) + H.c.,$$



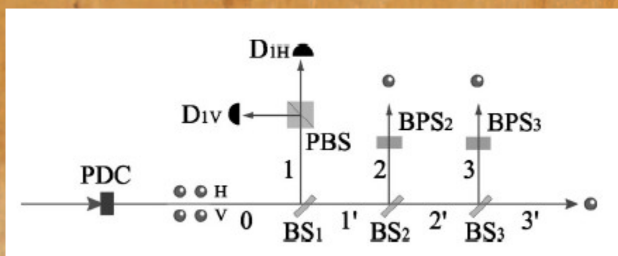
Kwiat *et al.*, Phys. Rev. Lett. 75, 4337 (1995)

$$|\Psi(t)\rangle = \exp(-it\hat{H}_\mathrm{I}/\hbar)\,|\Psi_0\rangle$$

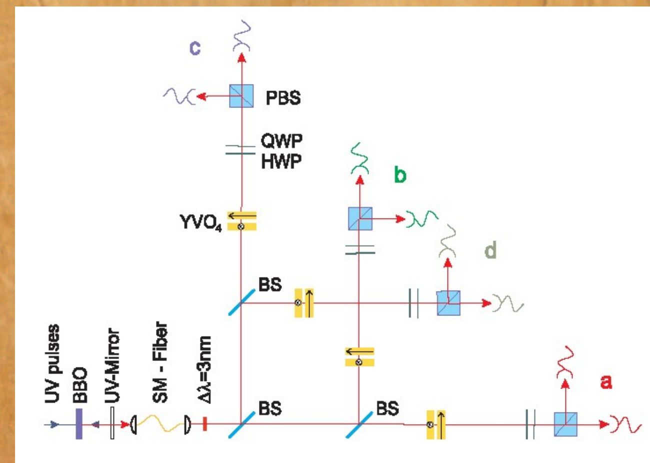
$$|\Psi(t)\rangle \approx [1 - it\hat{H}_\mathrm{I}/\hbar + \tfrac{1}{2}(-it\hat{H}_\mathrm{I}/\hbar)^2]|\Psi_0\rangle$$

$$|\Psi(t)\rangle = (1 - \mu^2/2)|0\rangle_\mathrm{s}|0\rangle_\mathrm{i} - i\mu|1\rangle_\mathrm{s}|1\rangle_\mathrm{i}$$

I. GENERATION

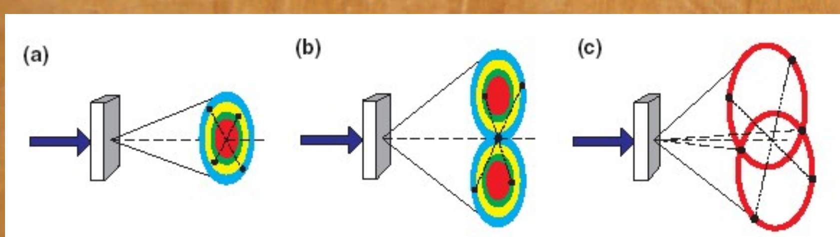


T. Yamamoto *et al.*, Phys. Rev. A 66, 064301 (2002)



N. Kiesel *et al.*, Phys. Rev. Lett. 98, 063604 (2007)

Y. Shih, Rep. Prog. Phys. 66, 1009 (2003)



PDC-I PDC-II collinear PDC-II noncollinear

$$H_I = \hbar\alpha a^\dagger_{H_s} a^\dagger_{V_i} + H.c.$$

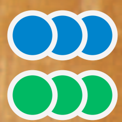
$$|\psi(t)\rangle = \mathrm{e}^{-itH_I/\hbar}\,|0\rangle_{H_s}|0\rangle_{V_i} = [1 - itH_I/\hbar + \frac{1}{2!}(-itH_I/\hbar)^2 + \ldots]\,|0\rangle_{H_s}|0\rangle_{V_i}$$

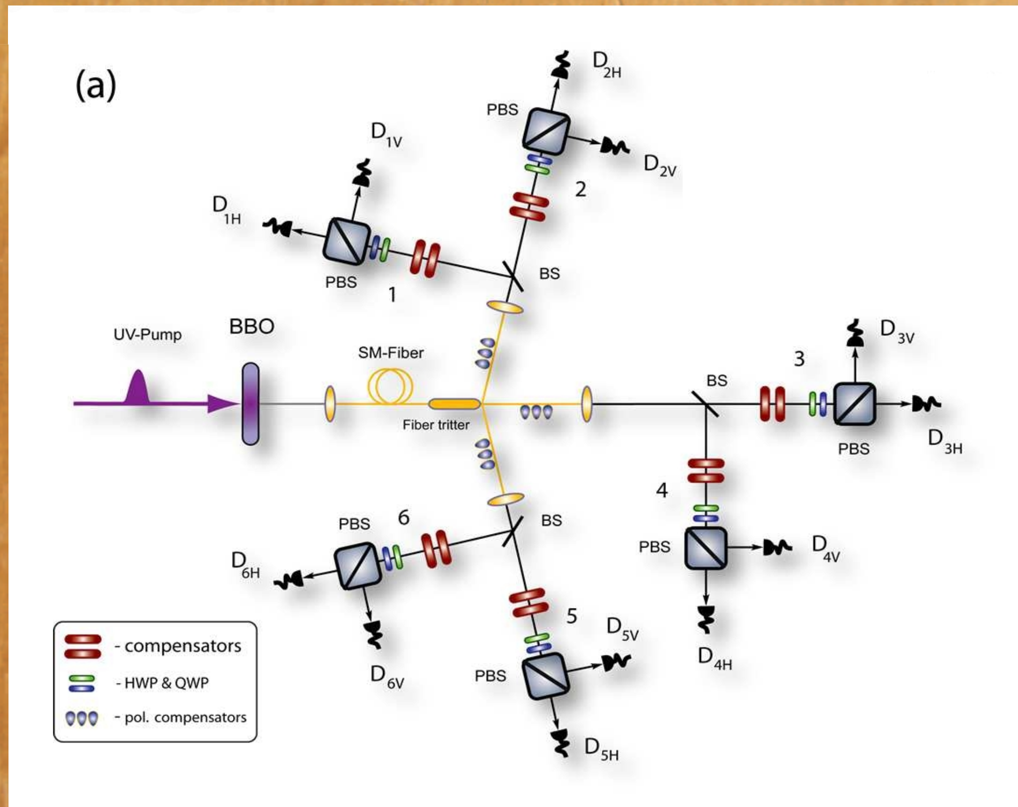
$$|\psi(t)\rangle = \mathcal{N}[|0\rangle_{H_s}|0\rangle_{V_i} - i\alpha\,|1\rangle_{H_s}|1\rangle_{V_i} - \alpha^2\,|2\rangle_{H_s}|2\rangle_{V_i} + i\alpha^3\,|3\rangle_{H_s}|3\rangle_{V_i} + \ldots]$$

I. GENERATION

$|3\rangle_{H_s}\,|3\rangle_{V_i}$

H
V



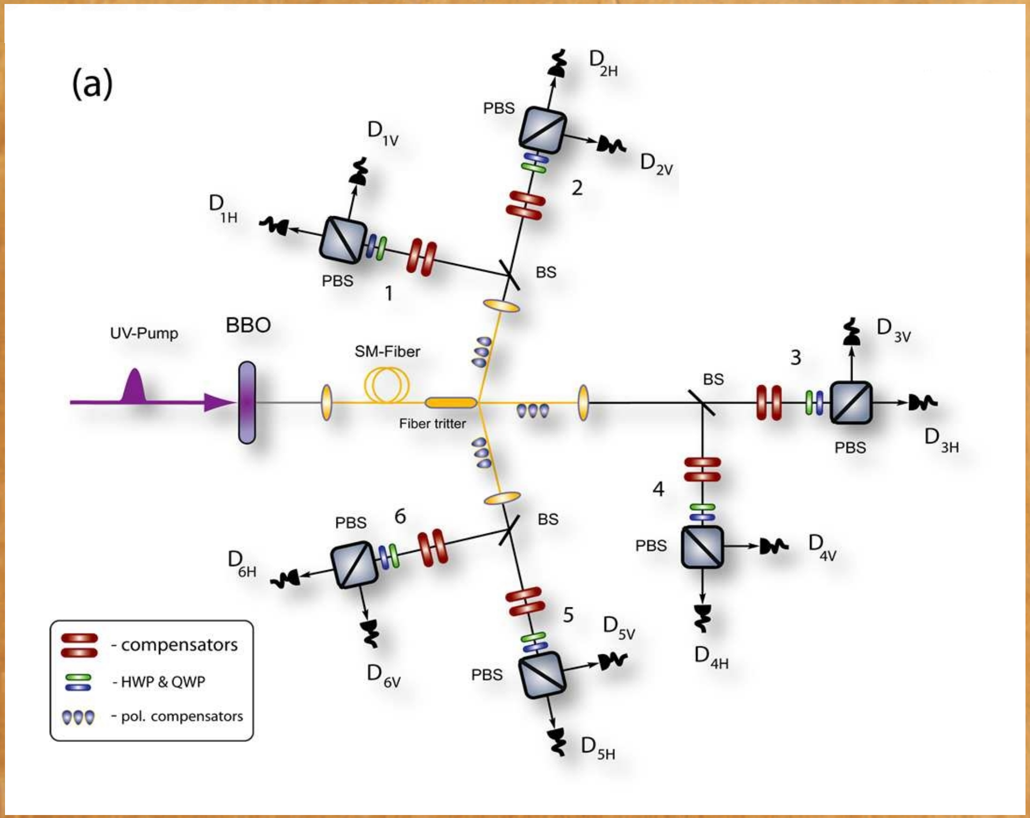


R. Prevedel *et al.*, Phys. Rev. Lett. 103, 020503 (2009)

$$
\begin{aligned}
|D_6^{(3)}\rangle = \frac{1}{\sqrt{20}}\Big[& |HHHVVV\rangle + |HHVVVH\rangle + |HHVVHV\rangle + |HHVHVV\rangle + |HVHVVH\rangle \\
& + |HVHVHV\rangle + |HVHHVV\rangle + |VHHVVH\rangle + |VHHVHV\rangle + |VHHHVV\rangle \\
& + |HVVHHV\rangle + |HVVHVH\rangle + |HVVVHH\rangle + |VHVHHV\rangle + |VHVHVH\rangle \\
& + |VHVVHH\rangle + |VVHHHV\rangle + |VVHHVH\rangle + |VVHVHH\rangle + |VVVHHH\rangle \Big]
\end{aligned}
$$

I. GENERATION

$|3\rangle_{H_s}|3\rangle_{V_i}$

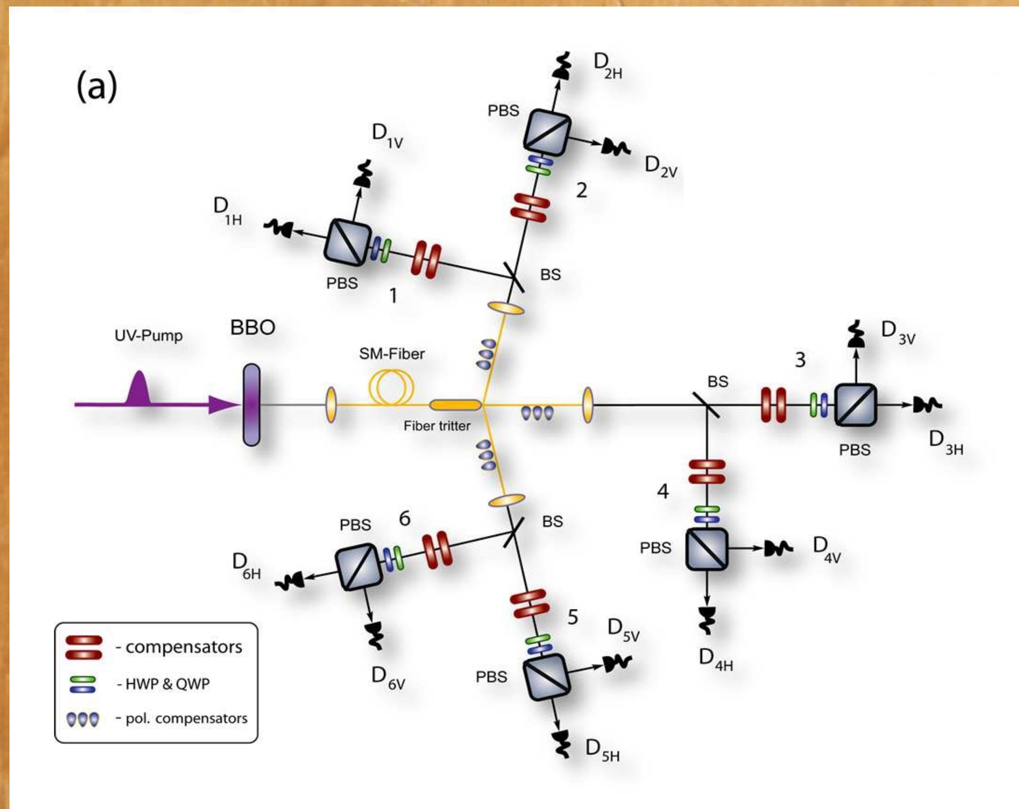


R. Prevedel *et al.*, Phys. Rev. Lett. 103, 020503 (2009)

$$
\begin{aligned}
|D_6^{(3)}\rangle = \frac{1}{\sqrt{20}}\Big[& |HHHVVV\rangle + |HHVVVH\rangle + |HHVVHV\rangle + |HHVHVV\rangle + |HVHVVH\rangle \\
& + |HVHVHV\rangle + |HVHHVV\rangle + |VHHVVH\rangle + |VHHVHV\rangle + |VHHHVV\rangle \\
& + |HVVHHV\rangle + |HVVHVH\rangle + |HVVVHH\rangle + |VHVHHV\rangle + |VHVHVH\rangle \\
& + |VHVVHH\rangle + |VVHHHV\rangle + |VVHHVH\rangle + |VVHVHH\rangle + |VVVHHH\rangle \Big]
\end{aligned}
$$

I. GENERATION

$|4\rangle_{H_s}\,|4\rangle_{V_i}$



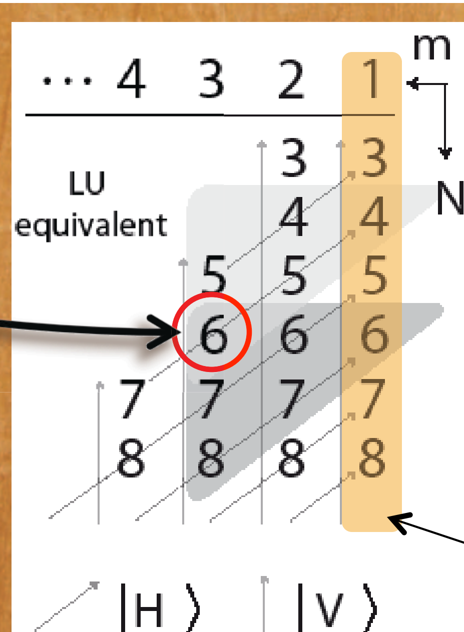
R. Prevedel *et al.*, Phys. Rev. Lett. 103, 020503 (2009)

$$
\begin{aligned}
|D_6^{(3)}\rangle = \frac{1}{\sqrt{20}}\Big[& |HHHVVV\rangle + |HHVVVH\rangle + |HHVVHV\rangle + |HHVHVV\rangle + |HVHVVH\rangle \\
& + |HVHVHV\rangle + |HVHHVV\rangle + |VHHVVH\rangle + |VHHVHV\rangle + |VHHHVV\rangle \\
& + |HVVHHV\rangle + |HVVHVH\rangle + |HVVVHH\rangle + |VHVHHV\rangle + |VHVHVH\rangle \\
& + |VHVVHH\rangle + |VVHHHV\rangle + |VVHHVH\rangle + |VVHVHH\rangle + |VVVHHH\rangle \Big]
\end{aligned}
$$

I. GENERATION

$$|D_6^{(3)}\rangle = \frac{1}{\sqrt{20}}\Big[|HHHVVV\rangle + |HHVVVH\rangle + |HHVVHV\rangle + |HHVHVV\rangle + |HVHVVH\rangle$$
$$+ |HVHVHV\rangle + |HVHHVV\rangle + |VHHVVH\rangle + |VHHVHV\rangle + |VHHHVV\rangle$$
$$+ |HVVHHV\rangle + |HVVHVH\rangle + |HVVVHH\rangle + |VHVHHV\rangle + |VHVHVH\rangle$$
$$+ |VHVVHH\rangle + |VVHHHV\rangle + |VVHHVH\rangle + |VVHVHH\rangle + |VVVHHH\rangle\Big]$$

$$|D_N^{(m)}\rangle = (C_N^m)^{-1/2}[(C_{N-1}^{m-1})^{1/2}\,|H\rangle\,|D_{N-1}^{(m-1)}\rangle + (C_{N-1}^{m})^{1/2}\,|V\rangle\,|D_{N-1}^{(m)}\rangle]$$



$$|D_N^{(m)}\rangle$$

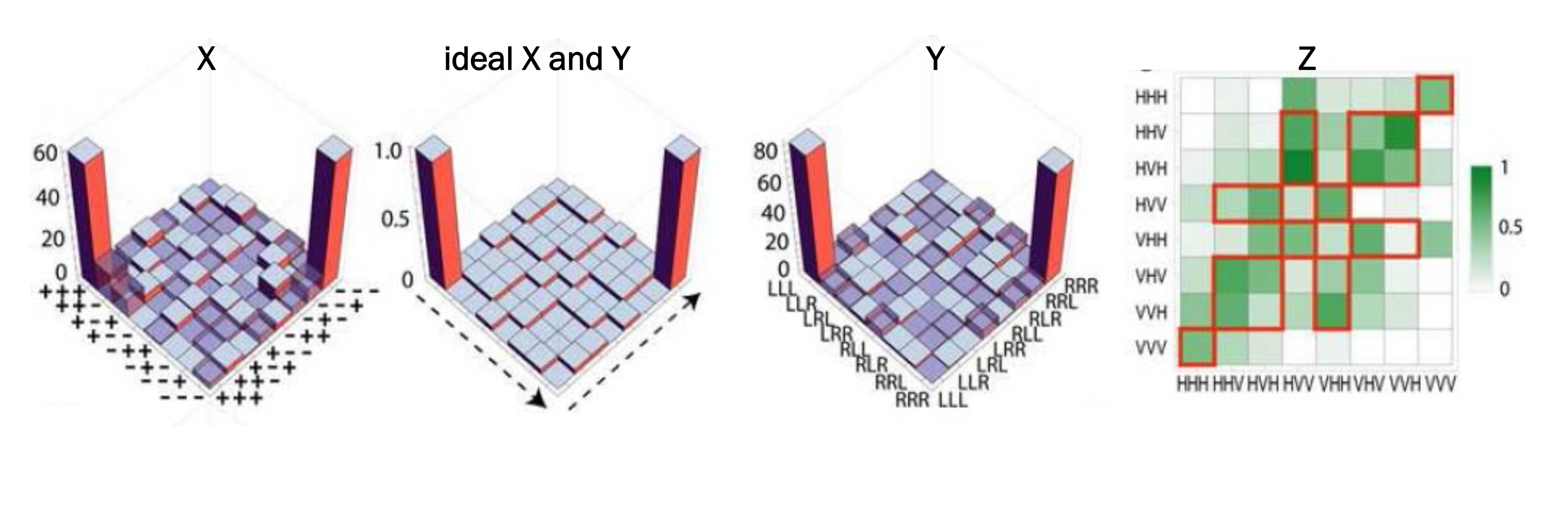
W states

Qubit 1, 2	State
H H	$\overline{W}$
H V V H	$D_4^{(2)}$
V V	W
P M M P	GHZ

II. CHARACTERISATION

II. CHARACTERISATION

$$|D_6^{(3)}\rangle = \frac{1}{\sqrt{20}} \sum_P |HHHVVV\rangle_{123456}$$



Pauli basis projective measurements

II. CHARACTERISATION

Genuine Multipartite Entanglement (GME) Witness:

$|\psi\rangle$

$$\mathcal{W} = \alpha \mathbb{1} - |\psi\rangle\langle\psi|,$$

$$\alpha = \max_{|\phi\rangle \in B} |\langle\phi|\psi\rangle|^2,$$

$\mathrm{Tr}(\mathcal{W}\rho_B) \geq 0$ $\quad$ $\mathrm{Tr}(\mathcal{W}|\psi\rangle\langle\psi|) < 0.$

M. Bourennane et al., Phys. Rev. Lett. 92, 087902 (2004)

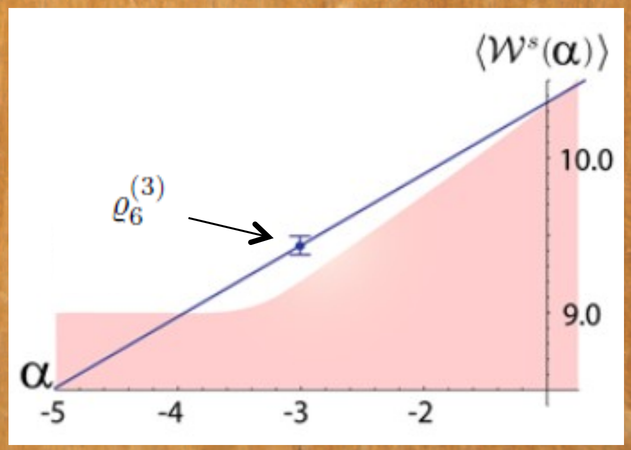
$$\langle \mathcal{W}^s \rangle_f = \langle J_x^2 + J_y^2 \rangle_f \qquad J_i = \tfrac{1}{2} \sum_{k=1}^{N} \sigma_i^{(k)} \quad (i = x, y, z)$$

G. Toth, J. Opt. Soc. Am. B 24 275 (2007)

$$\langle \mathcal{W}^s \rangle_{bs} \leq 11.02 \qquad \langle \mathcal{W}^s \rangle_f > 11.02 \qquad \langle \mathcal{W}^s \rangle_{\varrho_6^{(3)}} < 11.02$$

$$\mathcal{W}^s(\alpha) = J_x^2 + J_y^2 + \alpha J_z^2 \ (\alpha \in \mathbb{R}) \qquad \langle \mathcal{W}^s(\alpha) \rangle_{\varrho_6^{(3)}} > \langle \mathcal{W}^s(\alpha) \rangle_{bs}$$

$$\langle \mathcal{W}^s(\alpha) \rangle_{bs}^{\max} - \langle \mathcal{W}^s(\alpha) \rangle_{\varrho_6^{(3)}} = -0.24 \pm 0.06$$



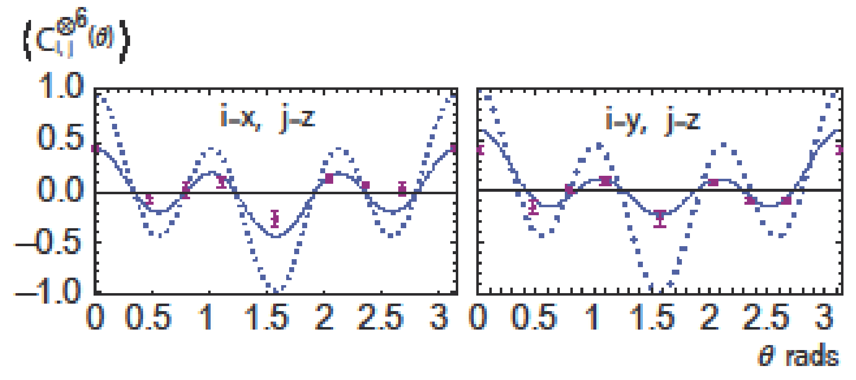
II. CHARACTERISATION

Correlations:

$$C_{i,j}^{\otimes N}(\theta) = \bigotimes_{k=1}^{N} \left(\cos\theta\sigma_i^{(k)} + \sin\theta\sigma_j^{(k)} \right)$$

$$\langle \mathcal{C}_{i,z}^{\otimes 6}(\theta) \rangle_{D_6^{(3)}} = [3\cos(2\theta) + 5\cos(6\theta)]/8$$

$$\langle D_n^{(n/2)} | \hat{\mathcal{C}}(\vartheta) | D_n^{(n/2)} \rangle = \sum_{k=0}^{n/2} (-1)^k (C_{n/2}^k)^2 (\cos\vartheta)^{n-2k} (\sin\vartheta)^{2k}$$



R. Prevedel *et al.*, Phys. Rev. Lett. 103, 020503 (2009)

S. Campbell *et al.*, New. J. Phys. 11, 073039 (2009)

Fidelity:

$$F_{D_6^{(3)}} = |D_6^{(3)}\rangle\langle D_6^{(3)}|$$

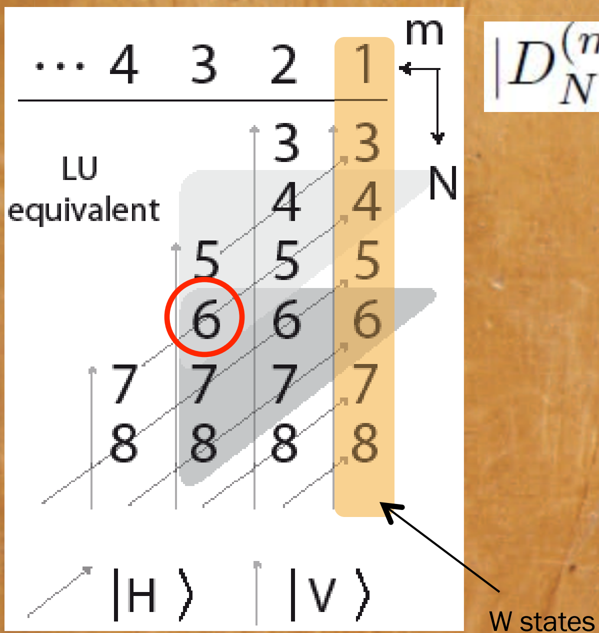
$$\frac{1}{2^N} \sum_{x_1,\ldots,x_N=0,x,y,z} T_{x_1\ldots x_N} \sigma_{x_1}^{(1)} \ldots \sigma_{x_N}^{(N)},$$

$$T_{x_1\ldots x_N} = \mathrm{tr}(\rho\sigma_{x_1}^{(1)} \ldots \sigma_{x_N}^{(N)}), \quad (x_i = x, y, z)$$

$$\langle F_{D_6^{(3)}} \rangle_{\varrho_6^{(3)}} = 0.56 \pm 0.02$$

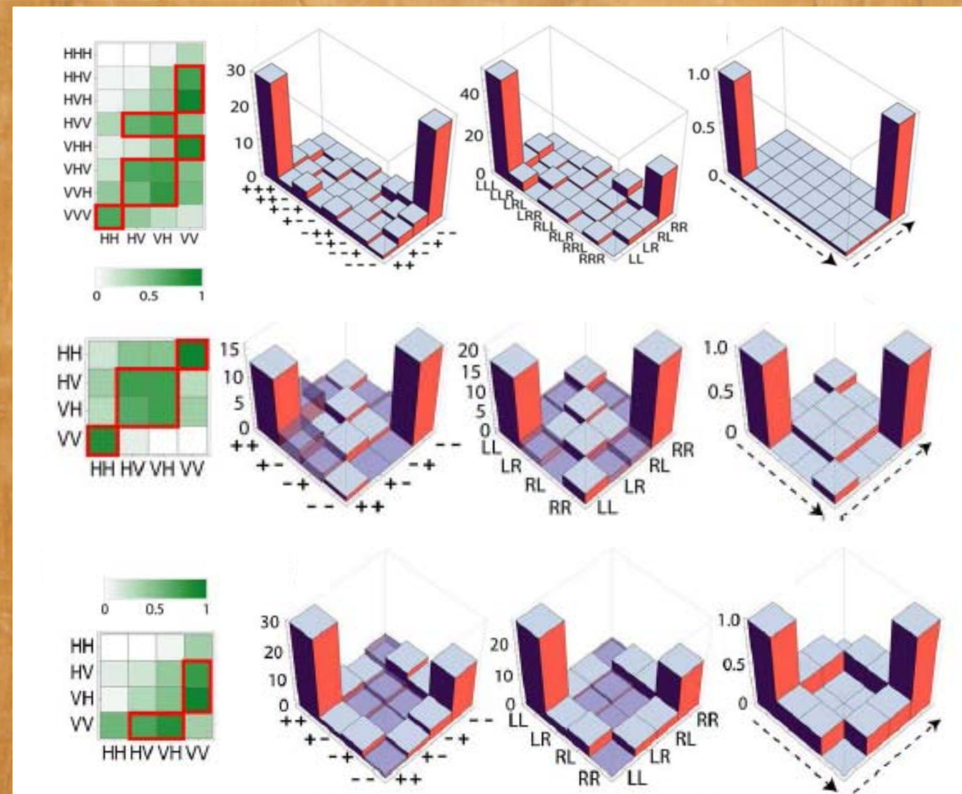
544 non-zero terms in T tensor
giving 21 local measurement settings

II. CHARACTERISATION



$|D_N^{(m)}\rangle$

Qubit 1, 2	State
H H	$\overline{W}$
H V V H	$D_4^{(2)}$
V V	W
P M M P	GHZ



$|D_5^{(2)}\rangle$

$|D_4^{(2)}\rangle$

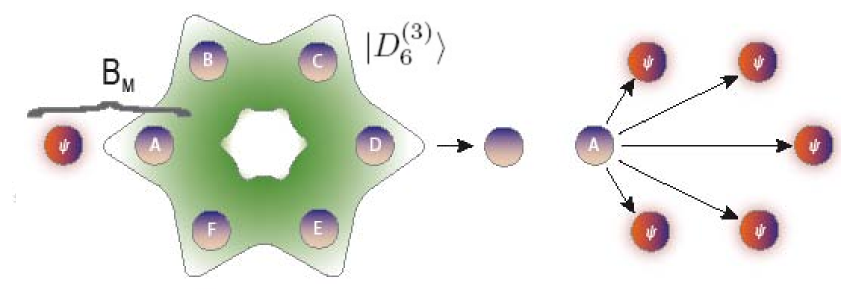
$|D_4^{(1)}\rangle$

	Witness	Fidelity
Dicke 5	-0.32±0.02	NA
Dicke 4	-0.16 ± 0.07	0.66 ± 0.05
W 4	-0.2 ± 0.1	0.62 ± 0.02
GHZ 4	-0.06 ± 0.02	0.56 ± 0.02

III. QUANTUM INFORMATION APPLICATIONS

III. QUANTUM INFORMATION APPLICATIONS

1. Open destination teleportation



$$|D_6^{(3)}\rangle = \frac{1}{\sqrt{20}} \sum_P |HHHVVV\rangle$$

Trace out all but 2 qubits-

$$\rho = \alpha_N |\psi^+\rangle\langle\psi^+| + (1-\alpha_N)[|HH\rangle\langle HH| + |VV\rangle\langle VV|]/2$$

$$\alpha_N = N/[2(N-1)]$$

$$|\psi^\pm\rangle = (|HV\rangle \pm |VH\rangle)/\sqrt{2}.$$

Maximal singlet fraction- $F_{\mathrm{msf}} = \alpha_N,$

Maximum fidelity for arbitrary state- $F_{\max} = (2F_{\mathrm{msf}} + 1)/3.$

$$\longrightarrow \quad F_{\max} = \frac{2N-1}{3(N-1)}.$$

ideal value 0.73

C.H. Bennett *et al.*, Phys. Rev. Lett. 70, 1895 (1993).
A. Karlsson and M. Bourennane, Phys. Rev. A 58, 4394 (1998).
M. Horodecki, P. Horodecki and R. Horodecki, Phys. Rev. A 60, 1888 (1999).
N. Kiesel *et al.*, Phys. Rev. Lett. 98, 063604 (2007).

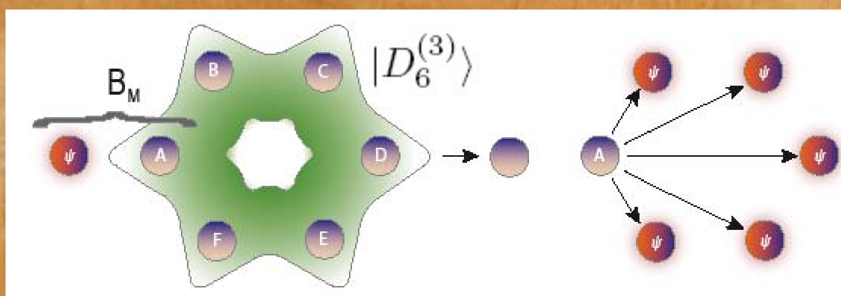
III. QUANTUM INFORMATION APPLICATIONS

1. Open destination teleportation

$$|D_6^{(3)}\rangle = \frac{1}{\sqrt{20}} \sum_P |HHHVVV\rangle$$

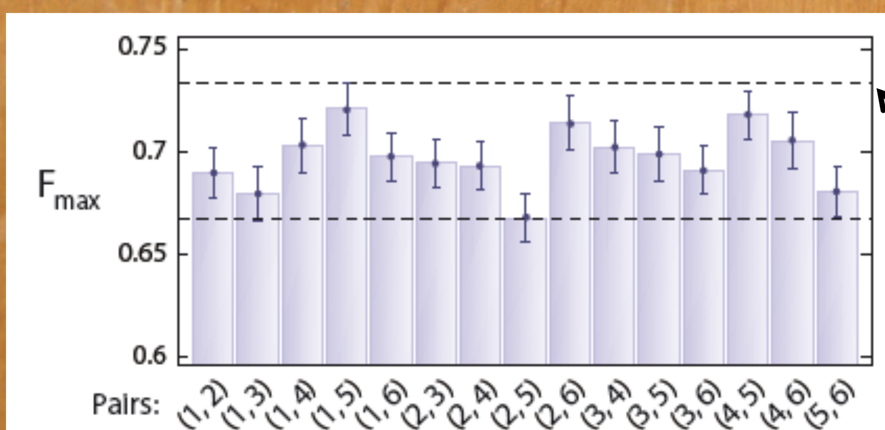
Perfect channel- $|H/V\rangle$

$p_s = \alpha_N \geq \frac{1}{2}$ $|\psi^+\rangle$



$$\bar{p}_s = 0.55 \pm 0.02$$

$$\langle \bar{F}_{\psi+} \rangle_{\rho_{exp}} = 0.71 \pm 0.02.$$



ideal value 0.73

$$F_{\max} = \frac{2N-1}{3(N-1)}.$$

III. QUANTUM INFORMATION APPLICATIONS

2. Telecloning

M. Murao *et al.*, Phys. Rev. A 59, 156 (1999).
V. Scarani *et al.*, Rev. Mod. Phys. 77, 1225 (2005).
N. Kiesel *et al.*, Phys. Rev. Lett. 98, 063604 (2007).

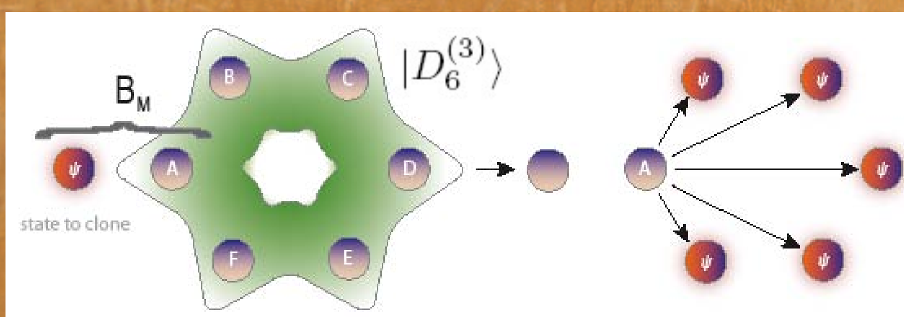
$$|D_6^{(3)}\rangle = \frac{1}{\sqrt{20}}\sum_P |HHHVVV\rangle$$

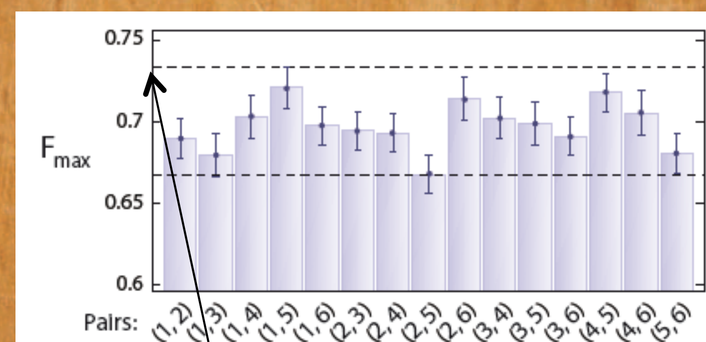
Trace out all but 2 qubits-

$$\rho = \alpha_N|\psi^+\rangle\langle\psi^+| + (1-\alpha_N)[|HH\rangle\langle HH| + |VV\rangle\langle VV|]/2$$

$$\alpha_N = N/[2(N-1)]$$

$$F_{\mathrm{msf}} = \alpha_N, \quad F_{\max} = (2F_{\mathrm{msf}}+1)/3.$$





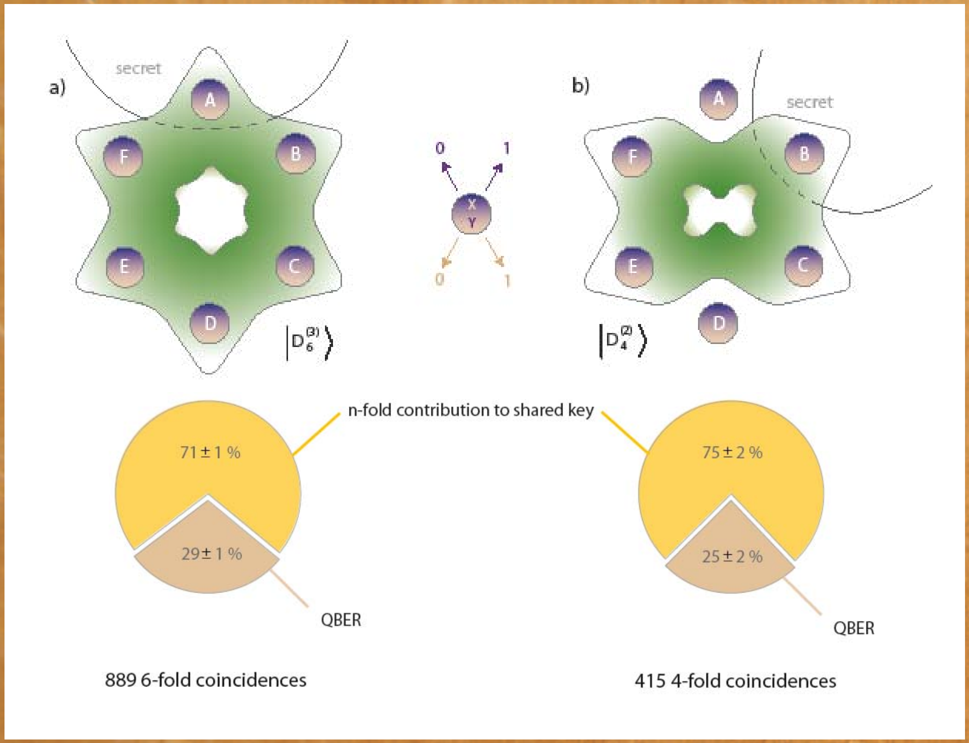
universal symmetric $1 \to (N-1)$ cloning

Upper limit

$$F_{\max} = \frac{2N-1}{3(N-1)}.$$

III. QUANTUM INFORMATION APPLICATIONS

3. Quantum secret sharing



$$|D_6^{(3)}\rangle = \frac{1}{\sqrt{20}} \sum_P |HHHVVV\rangle$$

$$\langle \sigma_{x,y}^{\otimes N} \rangle_{D_N^{(N/2)}} = 1$$

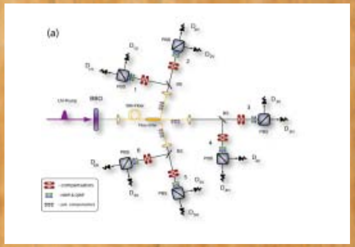
maximally-conjugate bases of $\sigma_{x,y}$

σ_x basis $\quad x_j \in \{0, 1\}$

$$x_1 = \oplus_{i=2}^{N} x_i$$

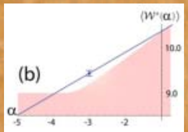
M. Hillery, V. Buzek and A. Berthiaume, Phys. Rev. A 59, 1829 (1999).
S. Gaertner *et al.*, Phys. Rev. Lett. 98, 020503 (2007).

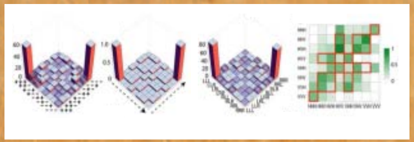
SUMMARY



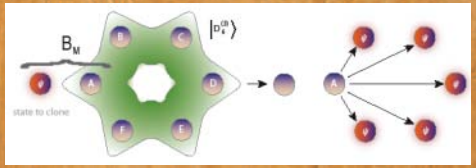
I. Generation of photonic Dicke states

II. Characterisation





III. Quantum information applications



THANK YOU FOR LISTENING!

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